Ocean Applications

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University of Maryland

2015 JCSDA Summer Colloquium

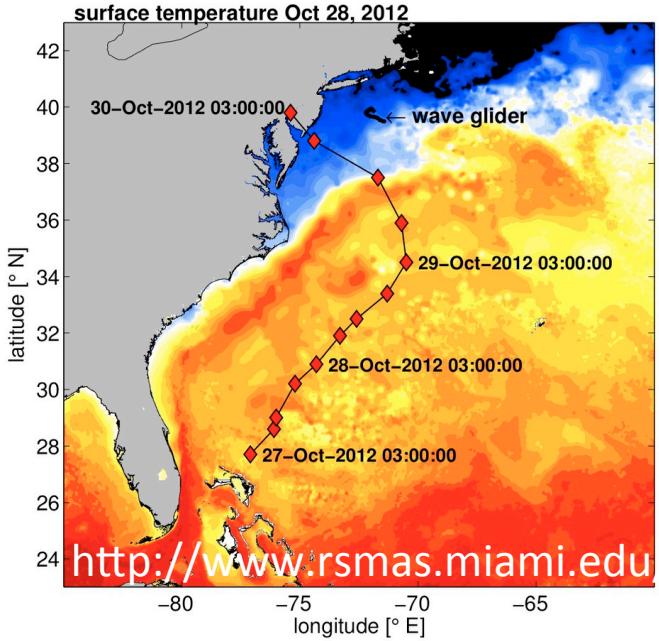
Outline

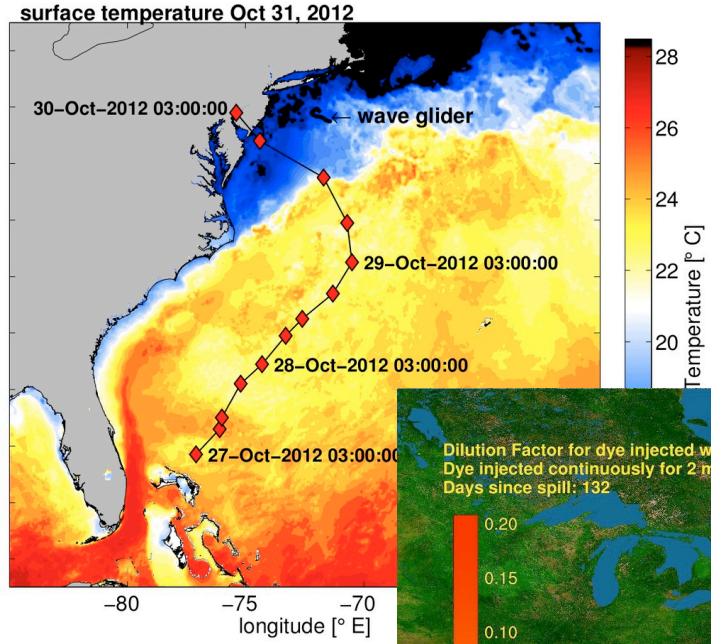
- Background with emphasis on operational oceanography
  - Why ocean data assimilation?
  - Ocean Modeling
  - Ocean Observations
  - Data Assimilation
  - Current State of Art
- Applications
  - Global ocean data assimilation
  - Regional data assimilation

- Concluding Remarks

Why Ocean Data Assimilation?

- Ocean is important, in many ways:
  - One of key players in the "big" earth system
  - One of key players of ecosystem & society





Operational Oceanography (OO)

- Started about 20yrs ago
- Goal:
  - Real-time monitoring and prediction, including
    - Currents, Temperature, Salinity
    - Primary production
    - Sea ice
  - Reanalysis
  - Targeted field campaign

Why it is a special topic, not a part of NWP?

- Ocean data assimilation is challenging
  - Observing system, spatially & temporally
    - Extremely inhomogeneous
    - Sparse
  - Modeling
    - Still lots of improvement
- Ocean data assimilation needs to
  - Face unique challenges
  - Should be adjusted to
    - » Make best use of observations
    - » Supplement modeling

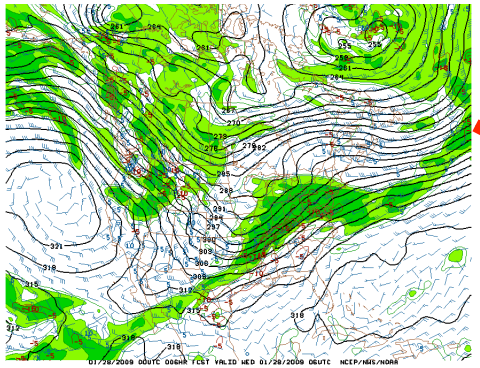
Schematic of Data Assimilation: Assimilation Window

- Data assimilation is a method that iterates the cycle over a window:

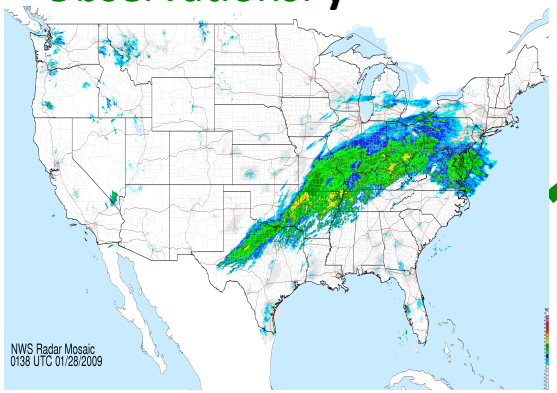
6hr for Operational System

- Model forecast: $\mathbf{x}^b$

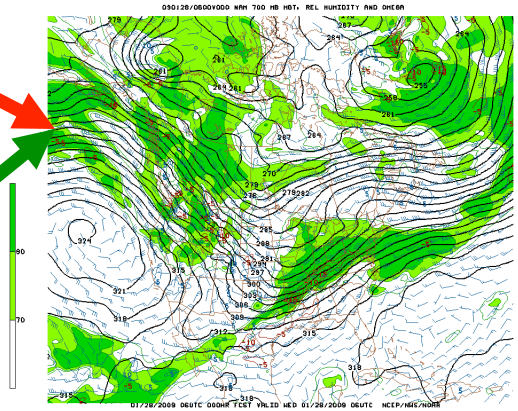
6h forecast (700mb) for 1.28.09 06UTC



- Observations: $\mathbf{y}^o$



- Analyzed model state: $\mathbf{x}^a$



Challenges of Data Assimilation: Model

$\mathbf{m}_{k,k-1}$ is
- nonlinear
- imperfect

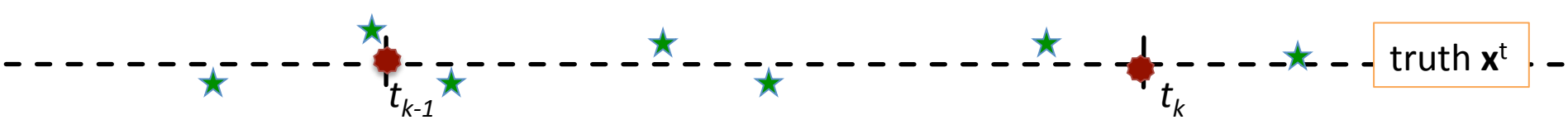
Assimilation cycle

Step 1. Model Forecast
Forecast→Background: $\mathbf{x}^b_k$
$\mathbf{x}_k = m_{k,k-1}(\mathbf{x}_{k-1})$

x can be large

Step 2. Assimilation
Integration of $\mathbf{x}^b_k$ and $\mathbf{y}^o_k$
→Analysis: $\mathbf{x}^a_k$ = func of $\mathbf{x}^b_k$ and $\mathbf{y}^o_k$

Observation
Measurement: $\mathbf{y}^o_{k'}$
$\mathbf{y}_{k'} = \mathbf{h}(\mathbf{x}_{k'})$



Challenges of Data Assimilation: Observation

Assimilation cycle

Step 1. Model Forecast
Forecast→Background: $\mathbf{x}^b_k$
$\mathbf{x}_k = m_{k,k-1}(\mathbf{x}_{k-1})$

$\mathbf{h}_{k'}$ is/may be
- nonlinear
- imperfect

Observation
Measurement: $\mathbf{y}^o_{k'}$
$\mathbf{y}_{k'} = \mathbf{h}(\mathbf{x}_{k'})$

$\mathbf{y}$ may be large or too small

Step 2. Assimilation
Integration of $\mathbf{x}^b_k$ and $\mathbf{y}^o_k$
→Analysis: $\mathbf{x}^a_k$ = func of $\mathbf{x}^b_k$ and $\mathbf{y}^o_k$

$\mathbf{y}_{k'}$ is/may be
- insufficient to determine $\mathbf{x}_k$
- not exactly at t_k

Data Assimilation Cycle With Probabilistic View

Assimilation cycle

Uncertainty evolution
$p(\mathbf{x}_k|\mathbf{Y}_{k-1}) = \boldsymbol{f}_{k,k-1}(p(\mathbf{x}_{k-1}|\mathbf{Y}_{k-1}))$

Step 1. Model Forecast
Forecast→Background: $\mathbf{x}^b_k$
$\mathbf{x}_k = m_{k,k-1}(\mathbf{x}_{k-1})$

Step 2. Assimilation
Integration of $\mathbf{x}^b_k$ and $\mathbf{y}^o_k$
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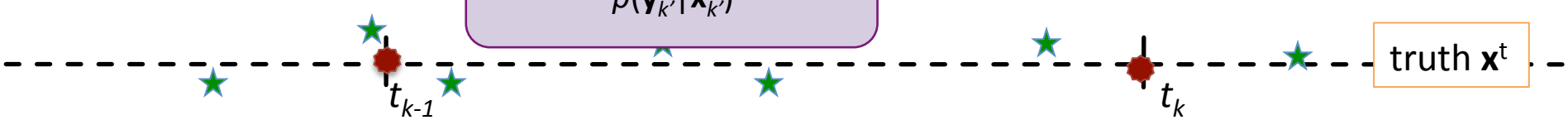
Observation
Measurement: $\mathbf{y}^o_{k'}$
$\mathbf{y}_{k'} = \mathbf{h}(\mathbf{x}_{k'})$

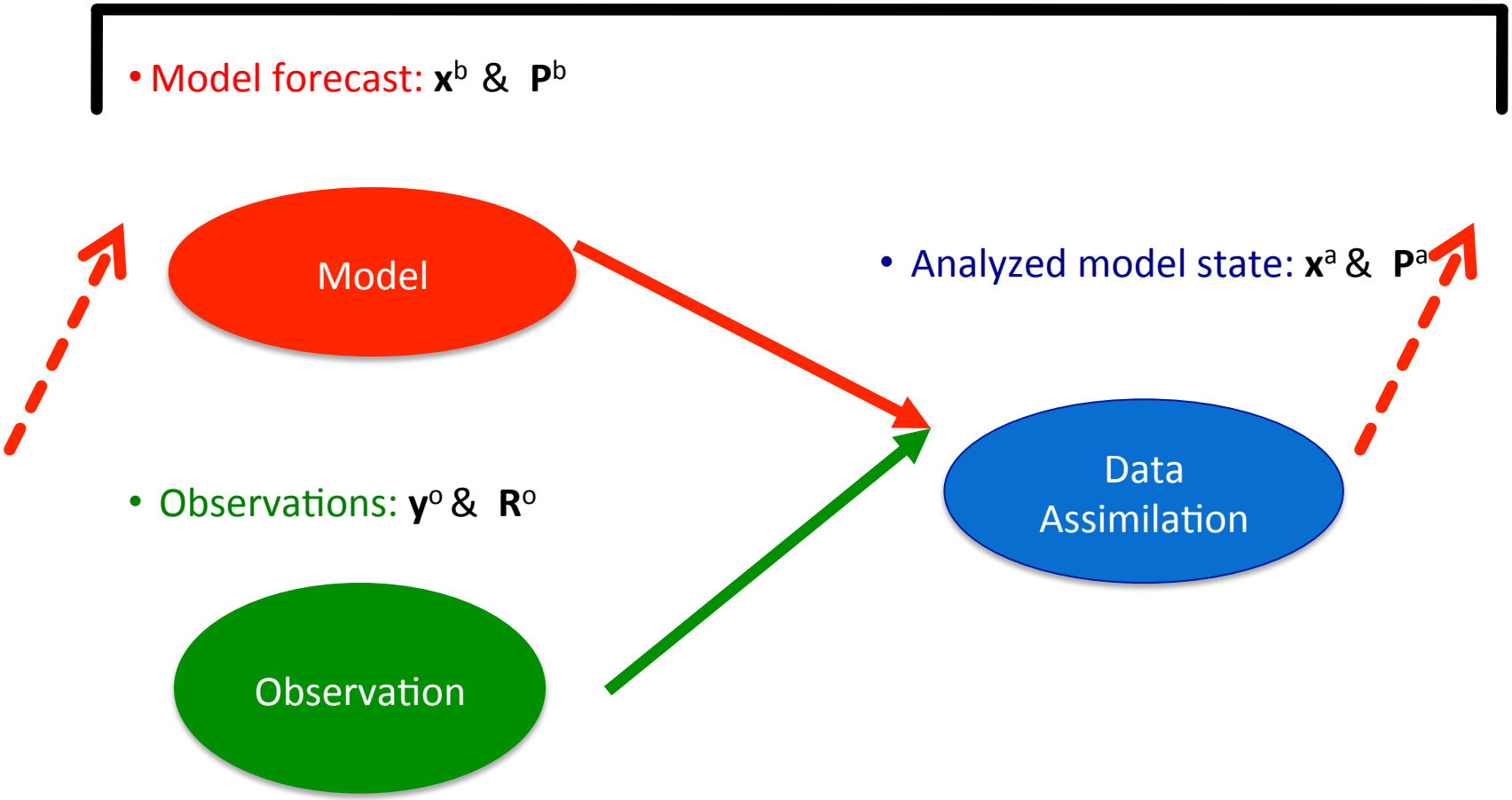
Uncertainty reduction/refinement
$p(\mathbf{x}_k|\mathbf{Y}_k) \leftarrow p(\mathbf{x}_{k-}|\mathbf{Y}_{k-1})$ & $p(\mathbf{y}_{k'}|\mathbf{x}_{k'})$
$\mathbf{Y}_k = \{\mathbf{y}_k, \mathbf{Y}_{k-1}\}$

Uncertainty in observation
$p(\mathbf{y}_{k'}|\mathbf{x}_{k'})$



Schematic of Data Assimilation: Assimilation Window

- Data assimilation is a iterative method that iterates the cycle over a window:



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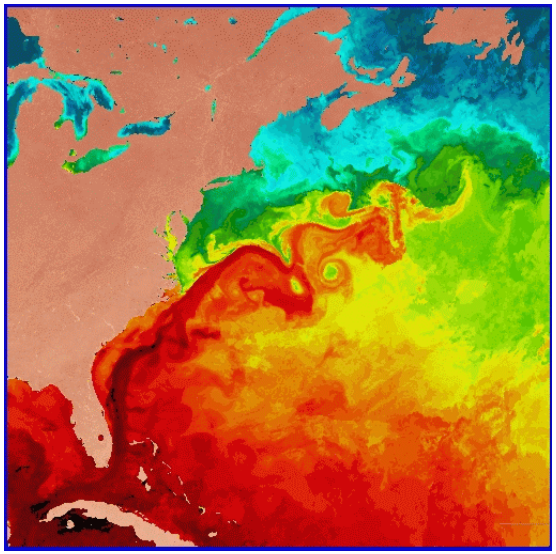
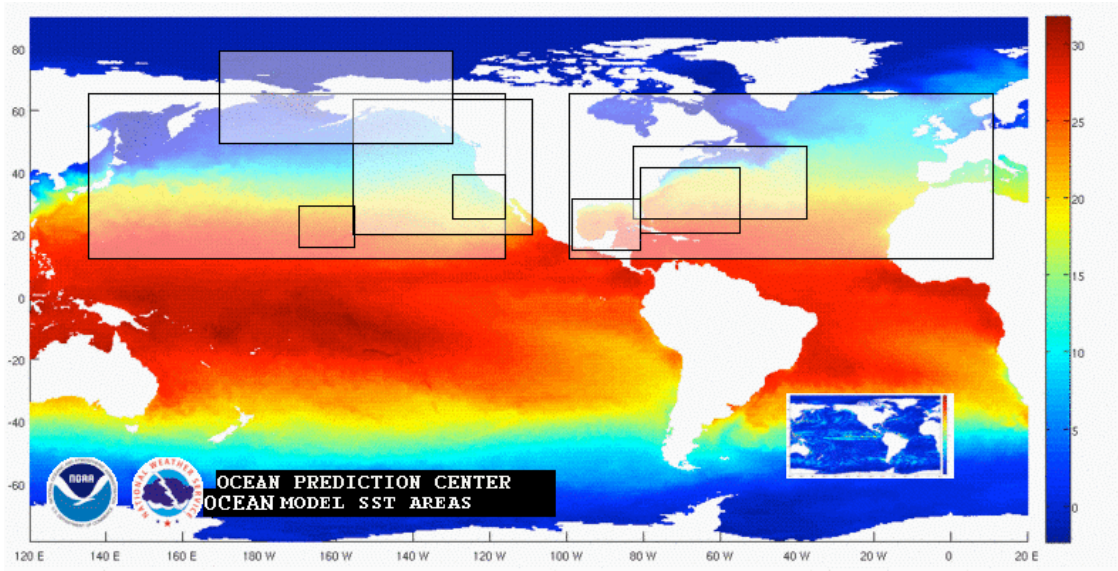
  - Reanalysis (global)
  - Targeted field campaign (regional)





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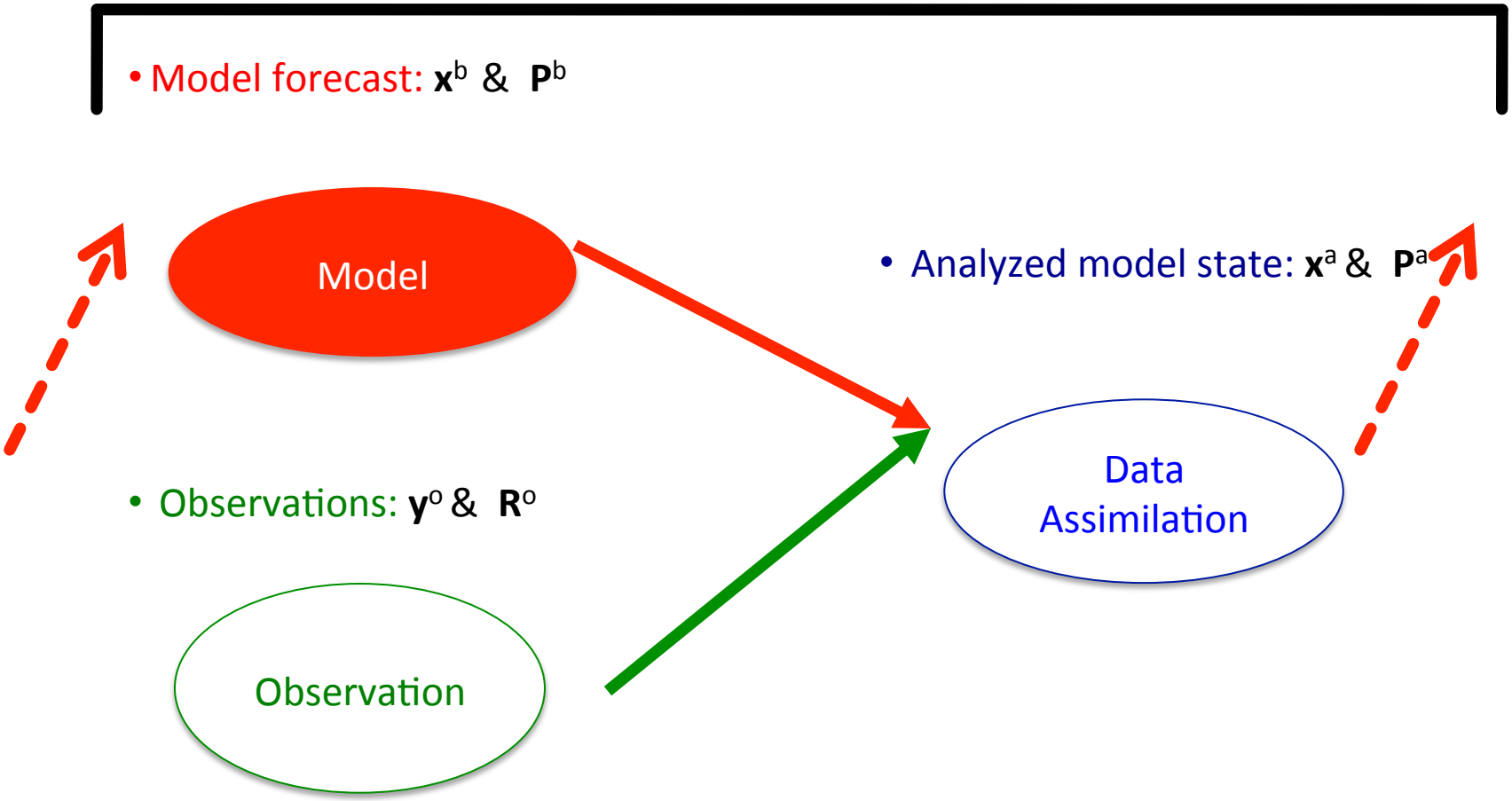


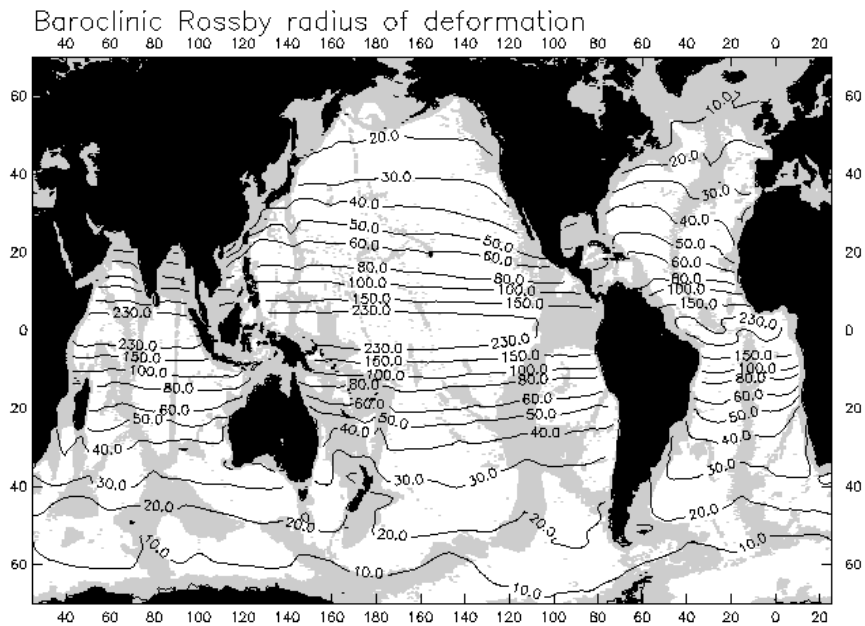
Ocean Modeling: Dynamics

- Horizontal scale
  - Baroclinic Rossby Def. Radius
    - 30km for ocean
    - 1000km for atmos

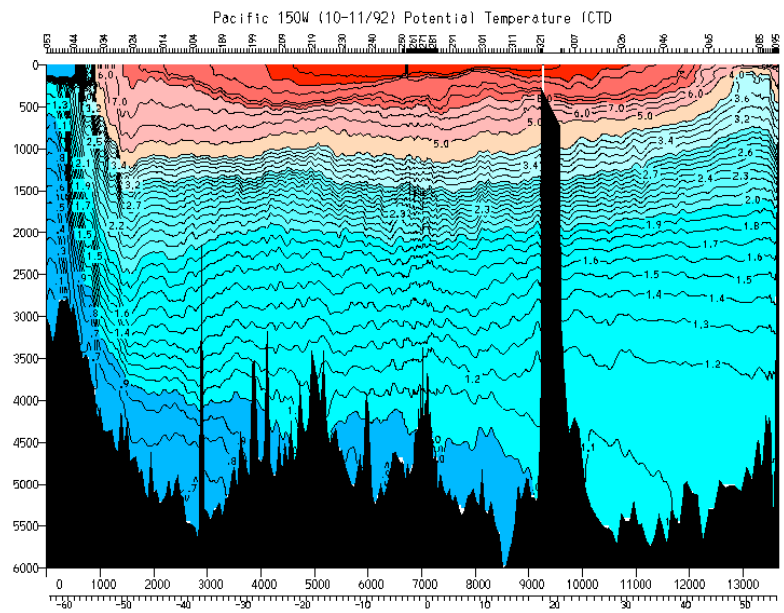
→ High resolution model

- Vertical Scale
  - High stratification
  - Most ocean obs are near surface

→ stratified information propagation



Baroclinic Rossby deformation radius



T distribution (Pacific)

Ocean Modeling: Dynamics

- Dynamical scales
  - Ocean: dynamics is highly nonlinear

Processes	Length Scale (km)	Time Scale	Velocity Scale (m/s)
Internal Waves	1-20	mins - hours	0.05-0.5
Coastal Upwelling	1-10	days	0.1-1
Large Eddies	10-200	days to weeks	0.1-1
Major Currents	50-500	weeks to seasons	0.5-2
Large-scale Circulation	10^3-10^4	decades-	0.01-0.1

  - Atmosphere

Processes	Length Scale (km)	Time Scale	Velocity Scale (m/s)
Sea Breeze	5-50	semi-diurnal	1-10
Mountain Waves	10-100	days	1-20
Synoptic Weather	100-5000	days to weeks	1-50
Prevailing Winds	global	seasons to years	5-50
Climate	global	decades-	1-50

Ocean Modeling

- Governing equations: primitive equation

$$\frac{\partial}{\partial t}u + u\frac{\partial}{\partial x}u + v\frac{\partial}{\partial y}u + w\frac{\partial}{\partial z}u = fv - \frac{1}{\rho}\frac{\partial}{\partial x}p + K\frac{\partial^2}{\partial z^2}u$$

$$\frac{\partial}{\partial t}v + u\frac{\partial}{\partial x}v + v\frac{\partial}{\partial y}v + w\frac{\partial}{\partial z}v = -fu - \frac{1}{\rho}\frac{\partial}{\partial y}p + K\frac{\partial^2}{\partial z^2}v$$

$$0 = -g - \frac{1}{\rho}\frac{\partial}{\partial z}p$$

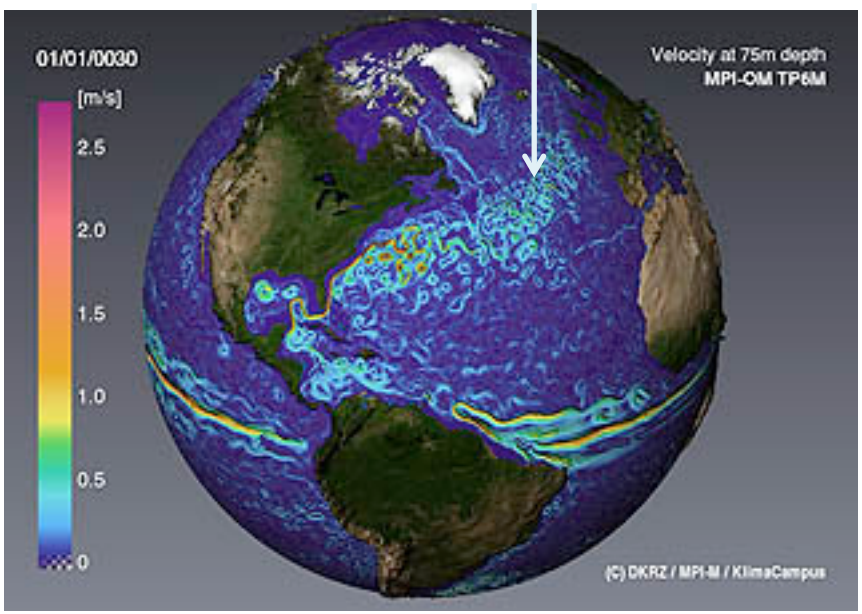
$$\nabla \cdot \mathbf{u} = 0$$

$$\rho = \rho(T, S)$$

$$\frac{d}{dt}T = \nabla \cdot (K_T \nabla T)$$

$$\frac{d}{dt}S = \nabla \cdot (K_S \nabla S)$$

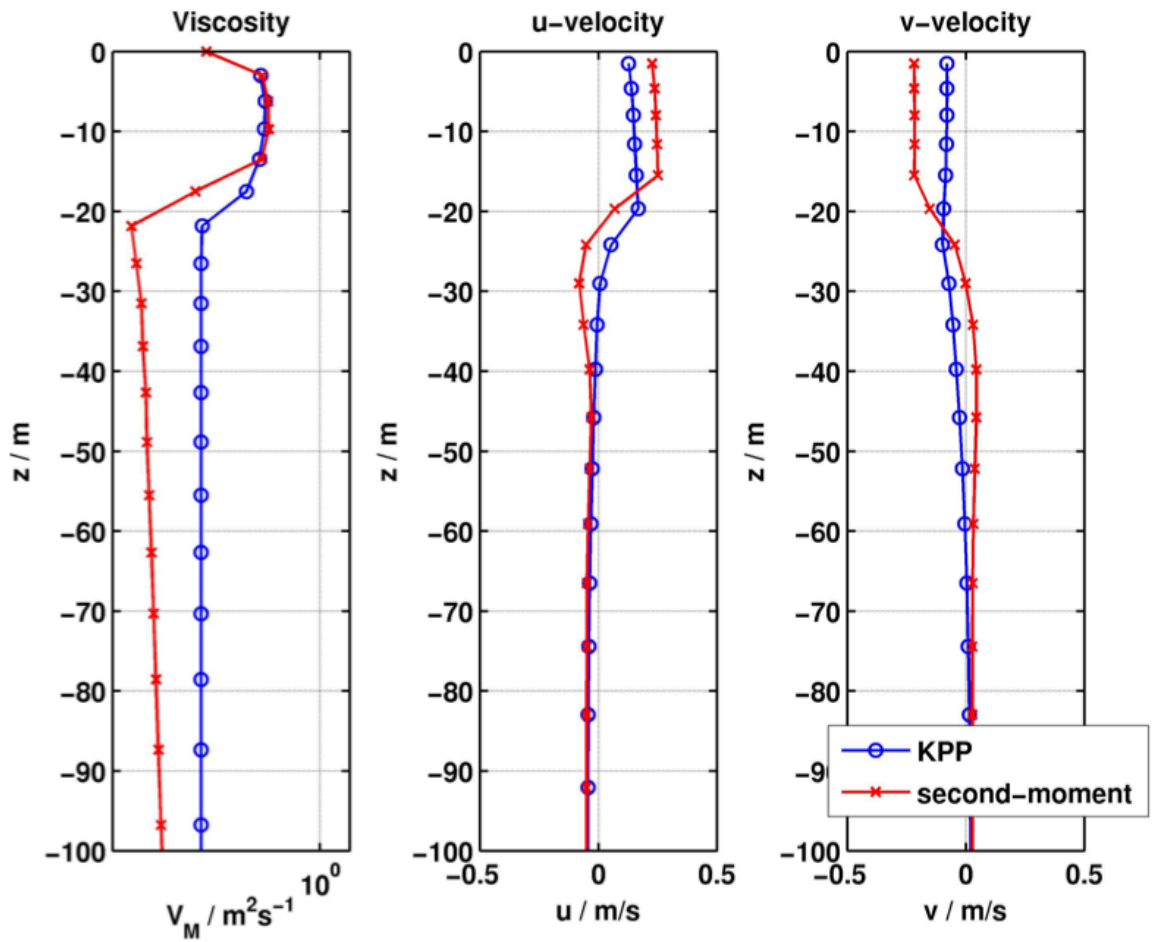
Nonlinearity at work



https://www.dkrz.de/Klimaforschung-en/konsortial-en/storm-en

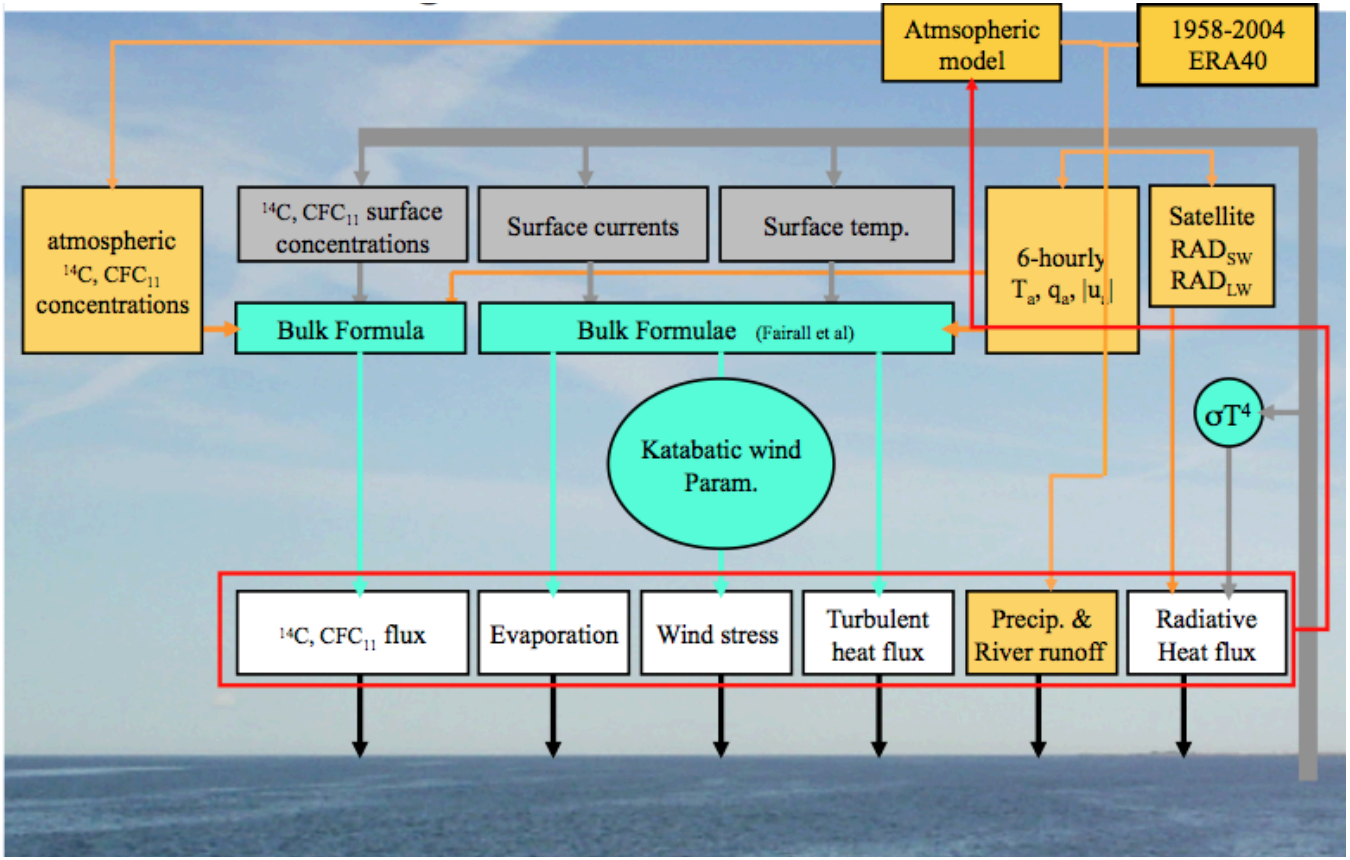
Ocean Modeling: Implementation

- Uncertainty by unresolved scales
  - Parameterization: Effect of dynamics at smaller scales than the grid



Ocean Modeling: Implementation

- Uncertainty by forcing
  - Example

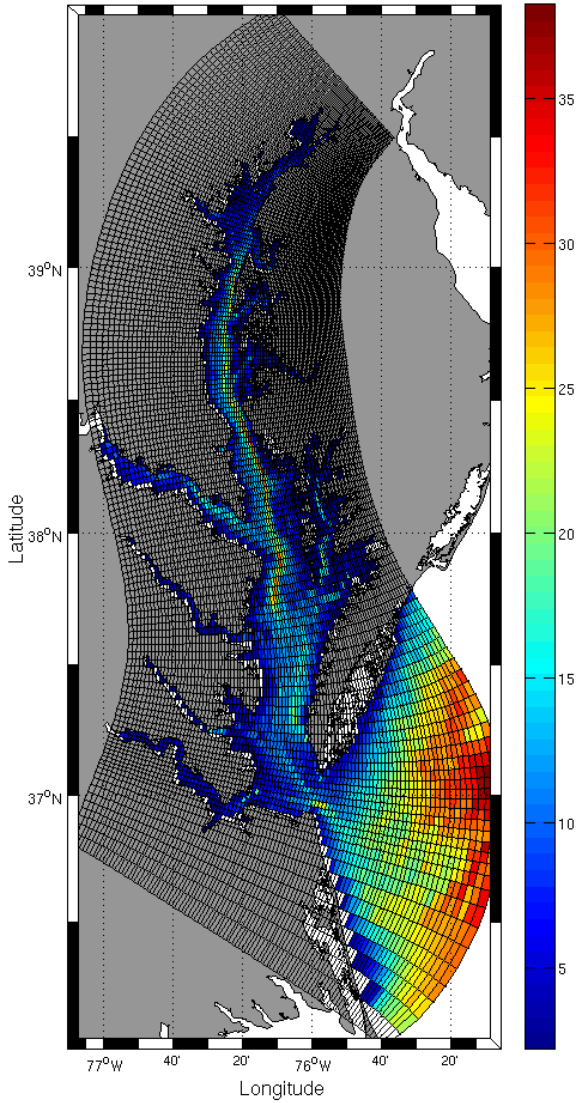


Sources

- Atmos
- Oceanic
- Physical
- Parameterization (in white)

Ocean Modeling: Implementation

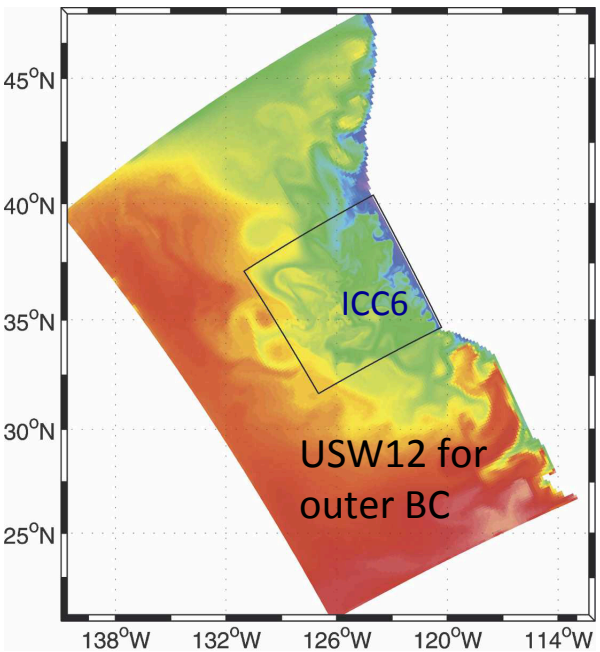
- Uncertainty by boundaries (physical choice, boundary conditions)
  - Example

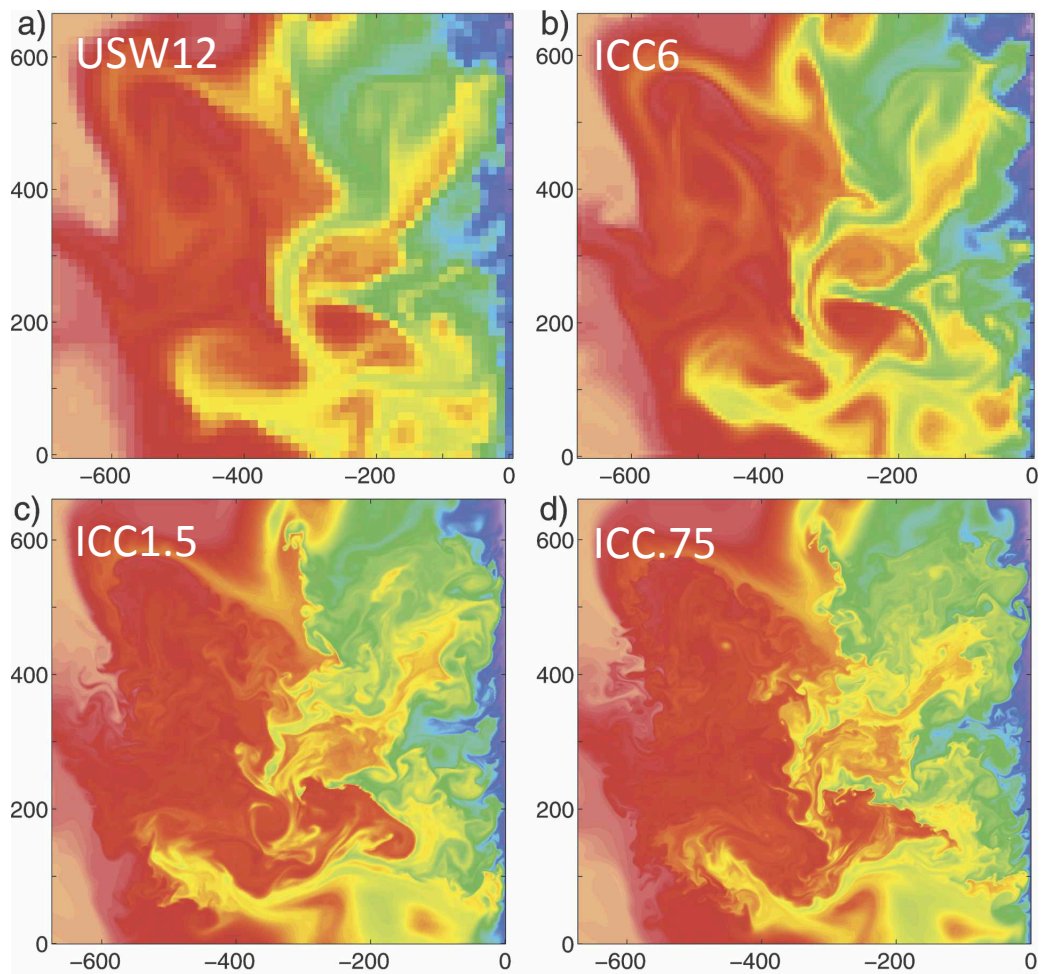


Hoffman etal (2012)

Ocean Modeling: Implementation

- Uncertainty by nesting for open boundary
  - Example





Capet etal (2008)

Ocean Modeling

- Ocean modeling: Still on-going

- For data assimilation, models
  - Nonlinear
  - Complex
  - Expensive
  - Subject to
    - Many uncertainties
    - Non-gaussianity

Schematic of Data Assimilation: Assimilation Window

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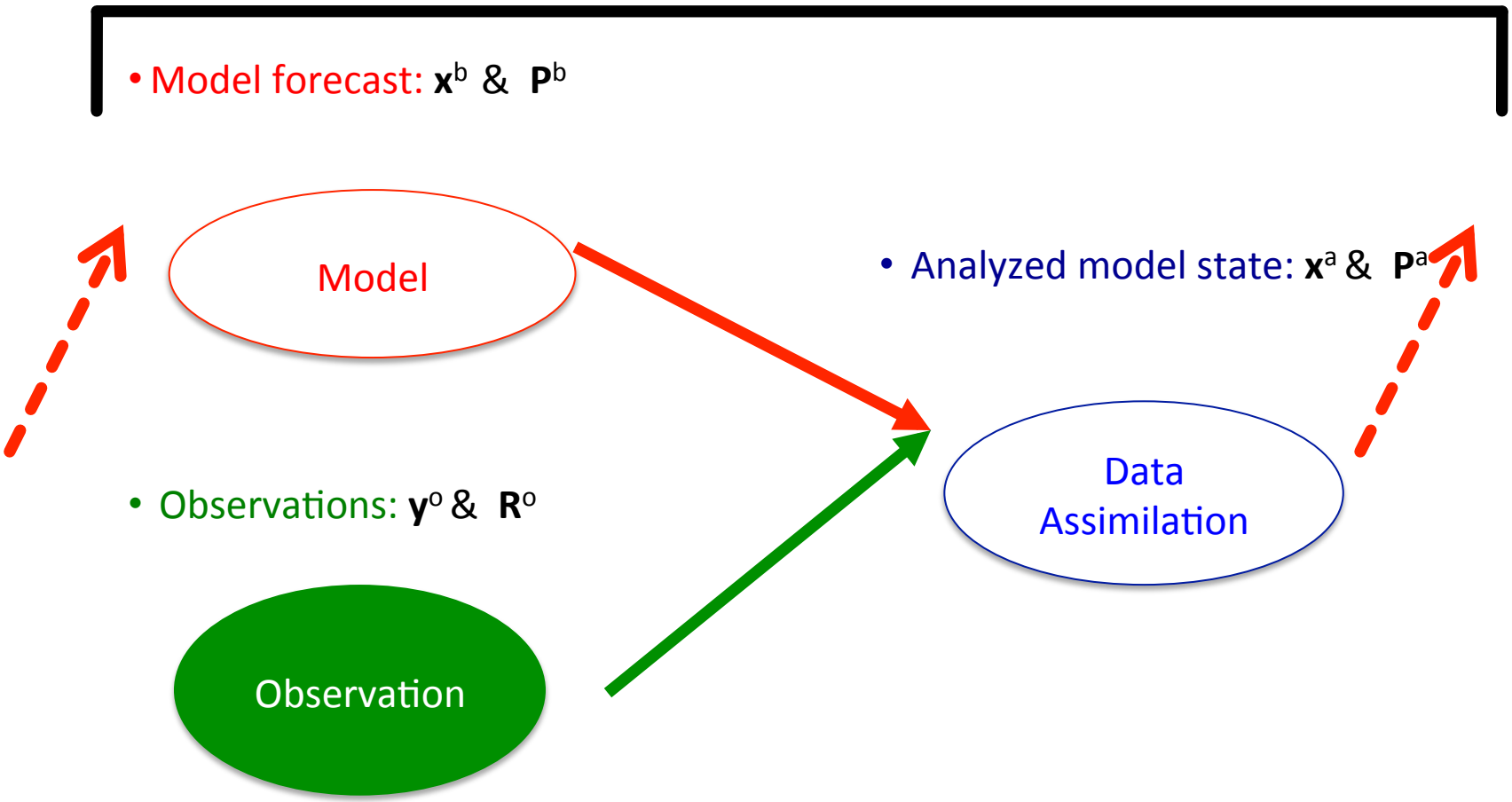
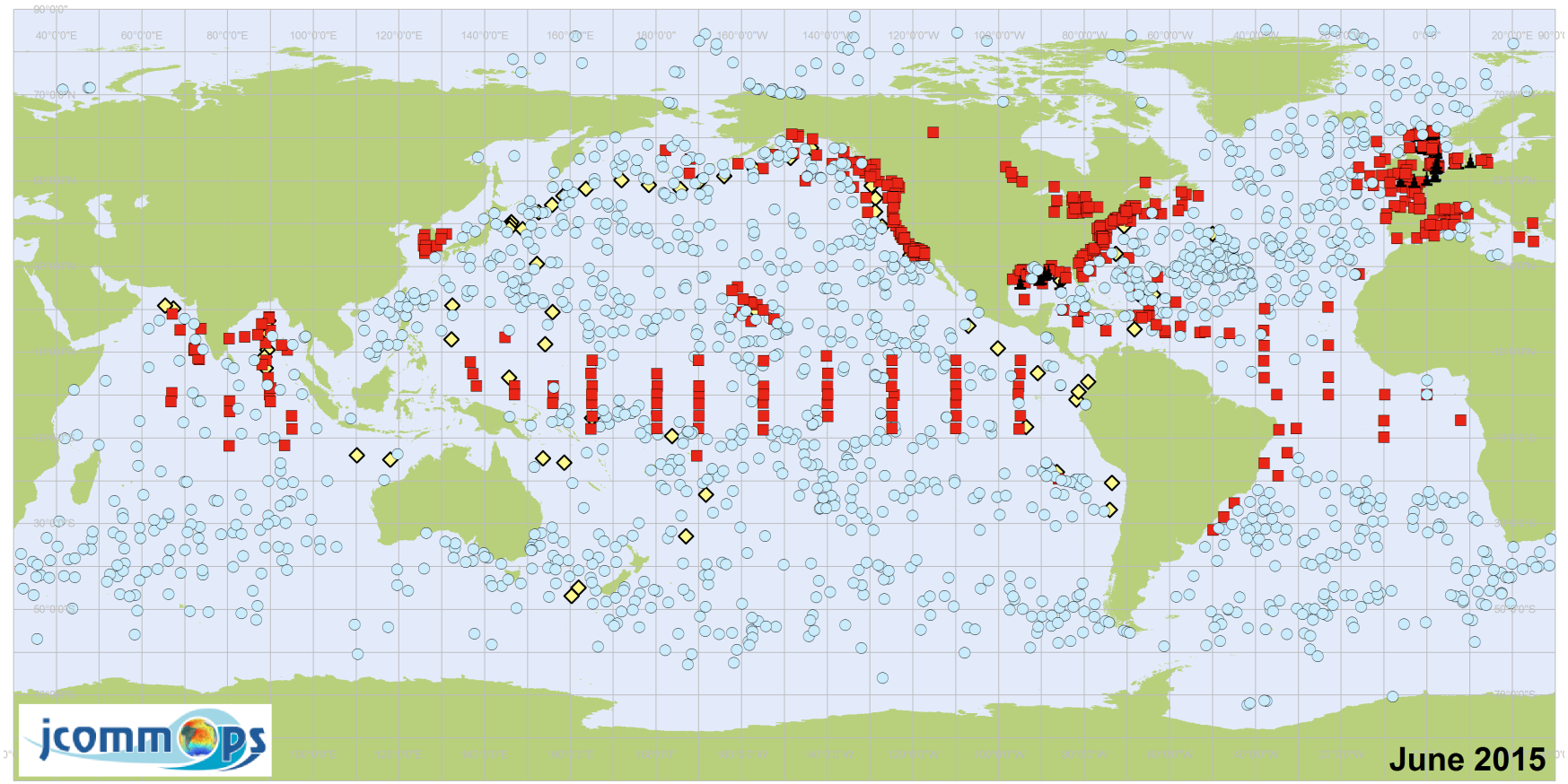


Ocean Observations

- In situ observation instruments
  - Profiling floats
  - Mooring:
  - Ships:
  - Surface drifters
  - Gliders
  - Marine mammals
- Satellite observation types
  - Altimetry
  - Sea surface temperature
  - Ocean color
- Other remote sensing
  - Surface current

Ocean Observations

- JCOMM In situ Observing Platform Support Centre (JCOMMOPS) for 1mo

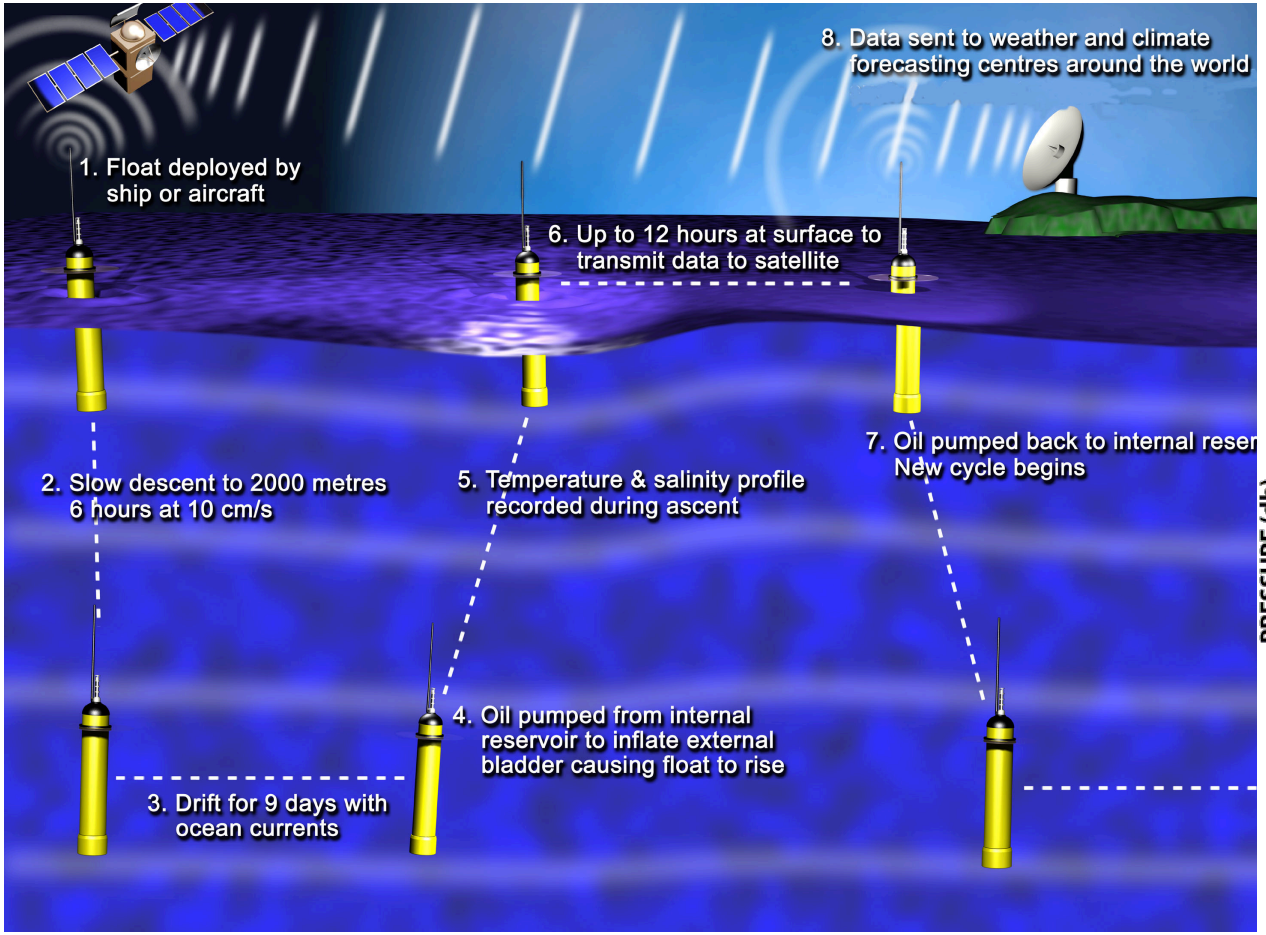


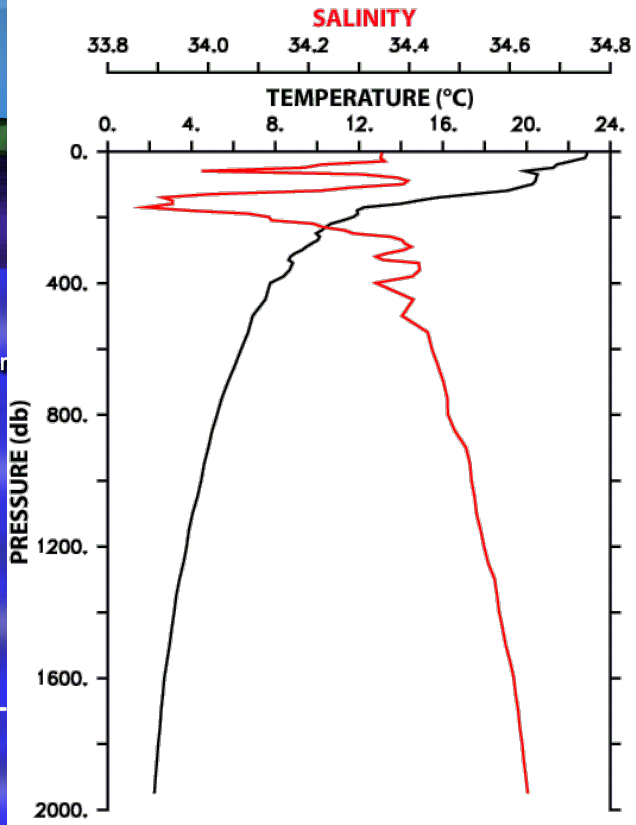
Drifting Buoys (1499) ■ Moored Buoys (470) ▲ Fixed Platforms (43) ◆ Tsunameter Buoys (62)

http://www.jcommops.org/new/

Ocean Observations

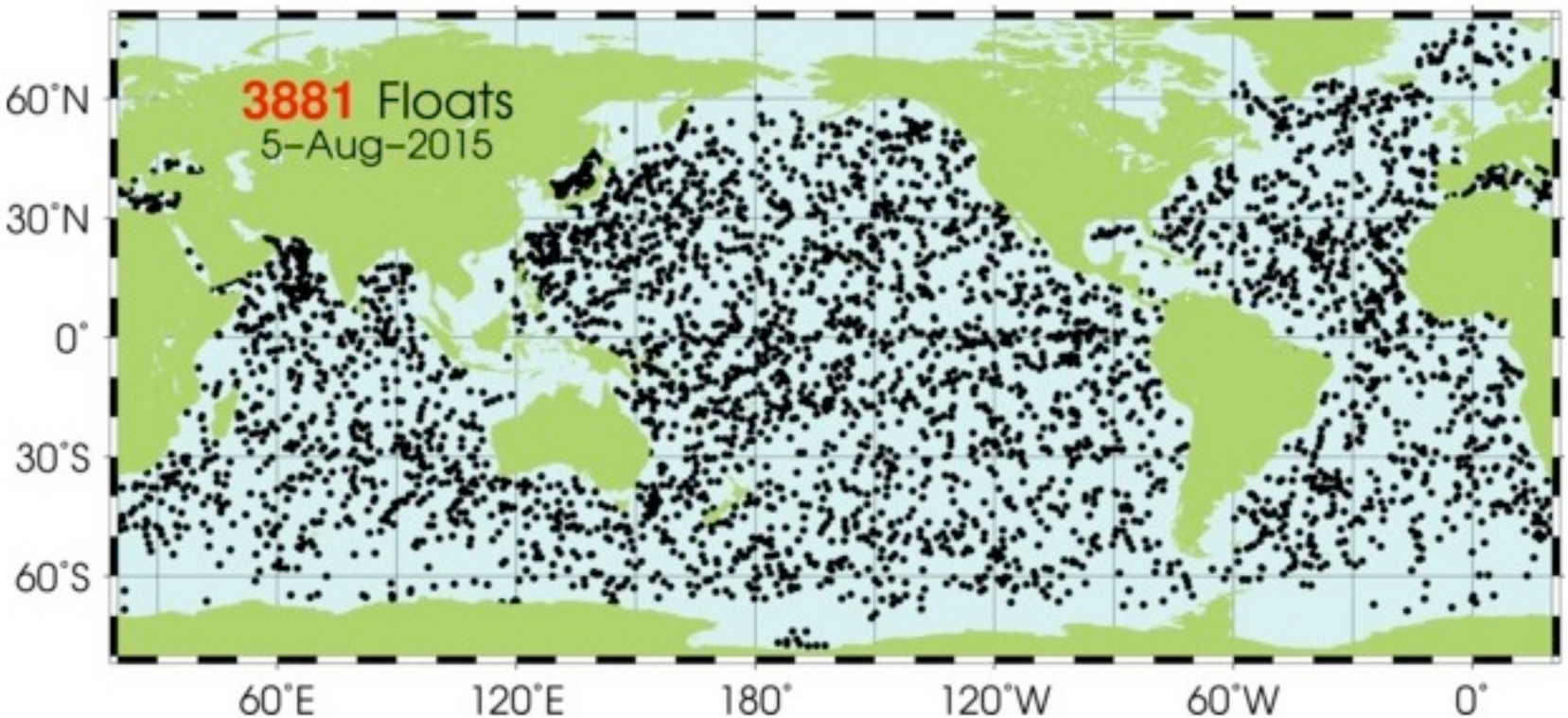
- In situ: ARGO





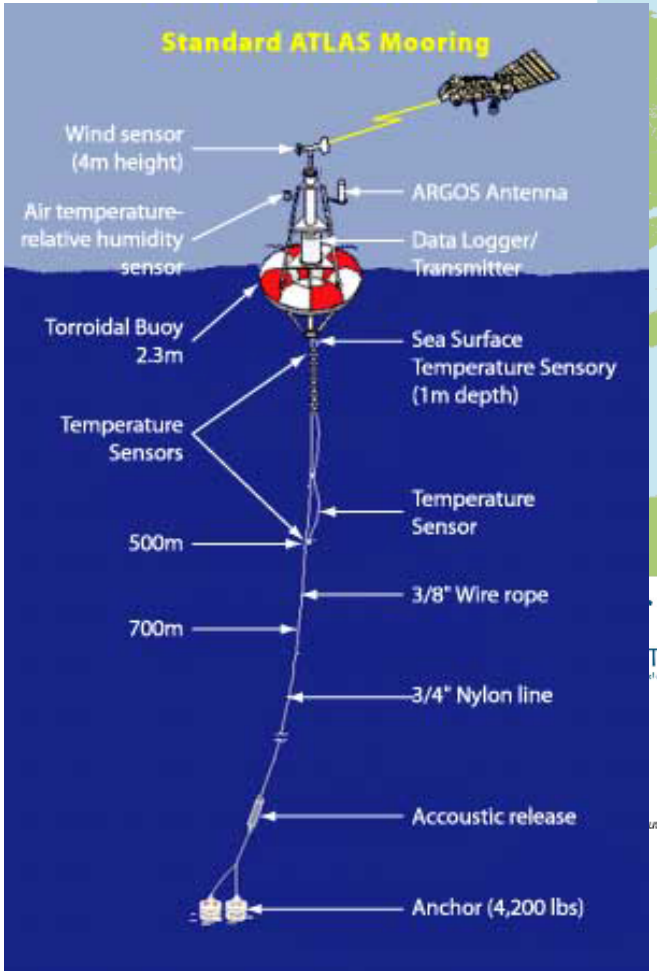
Ocean Observations

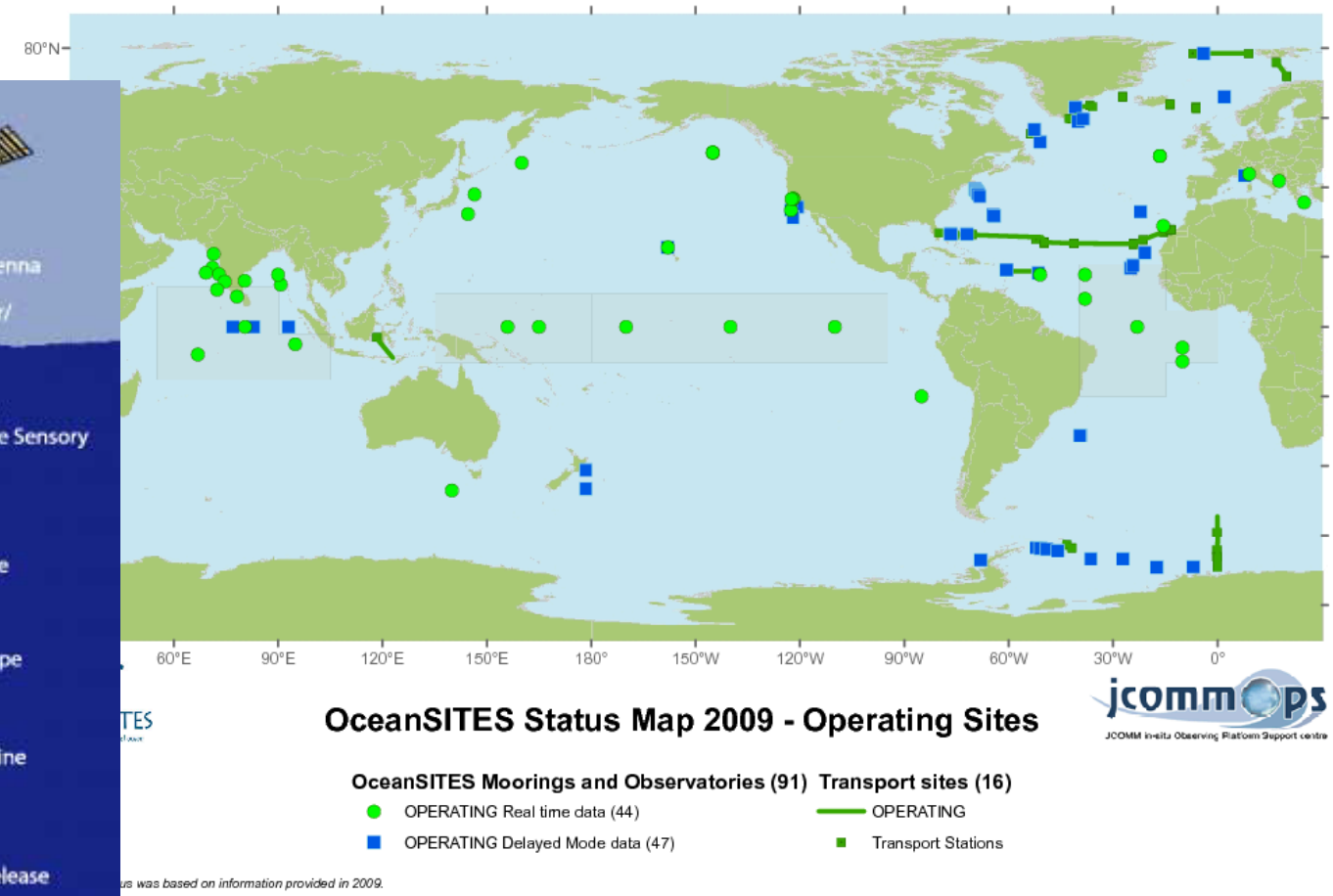
- In situ: ARGO



Ocean Observations

- In situ: Mooring

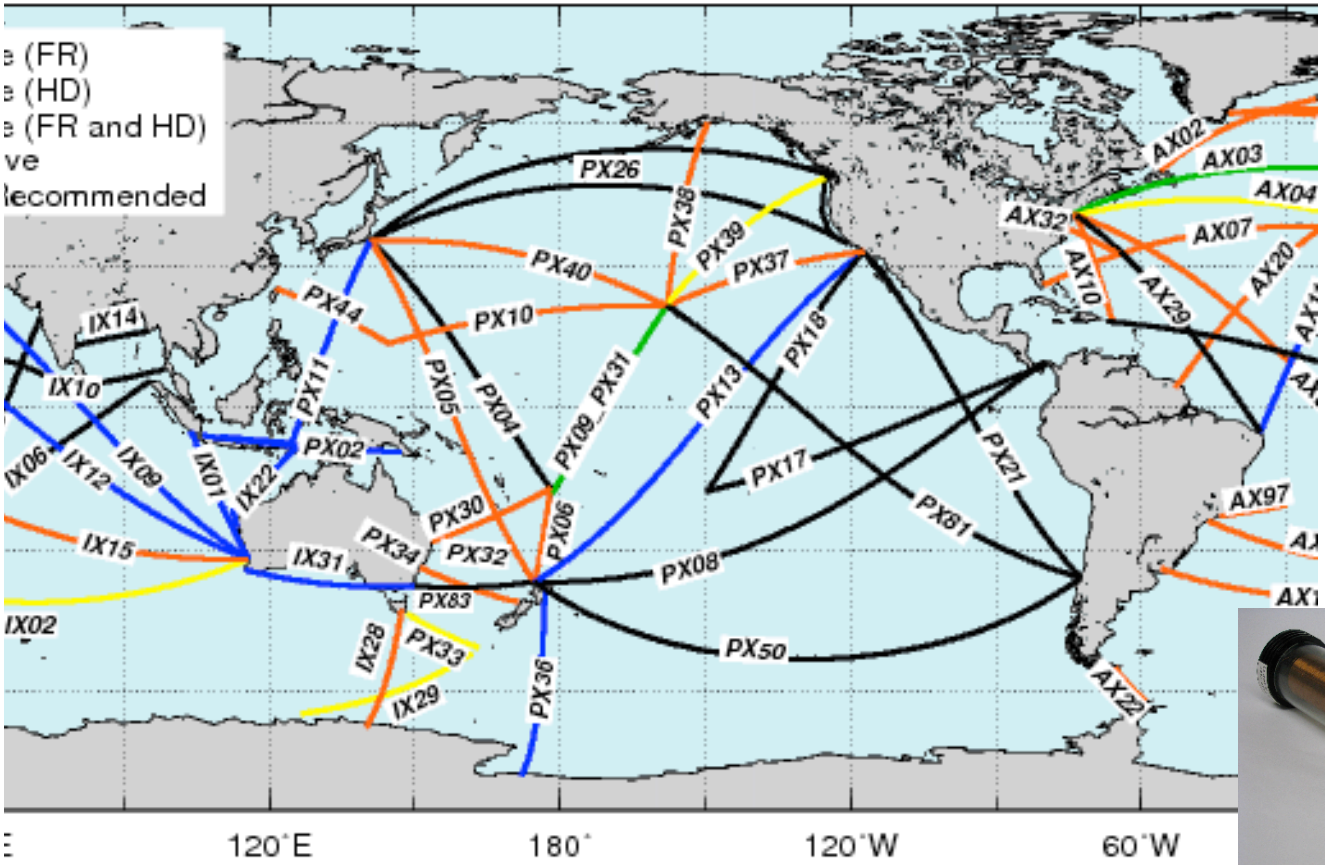


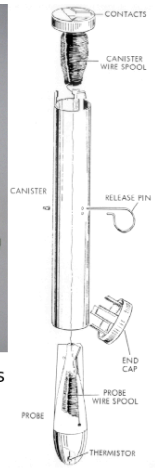


Ocean Observations

- In Situ: Ships of Opportunity Program

XBT Network Status and Implementation (2011–2012)



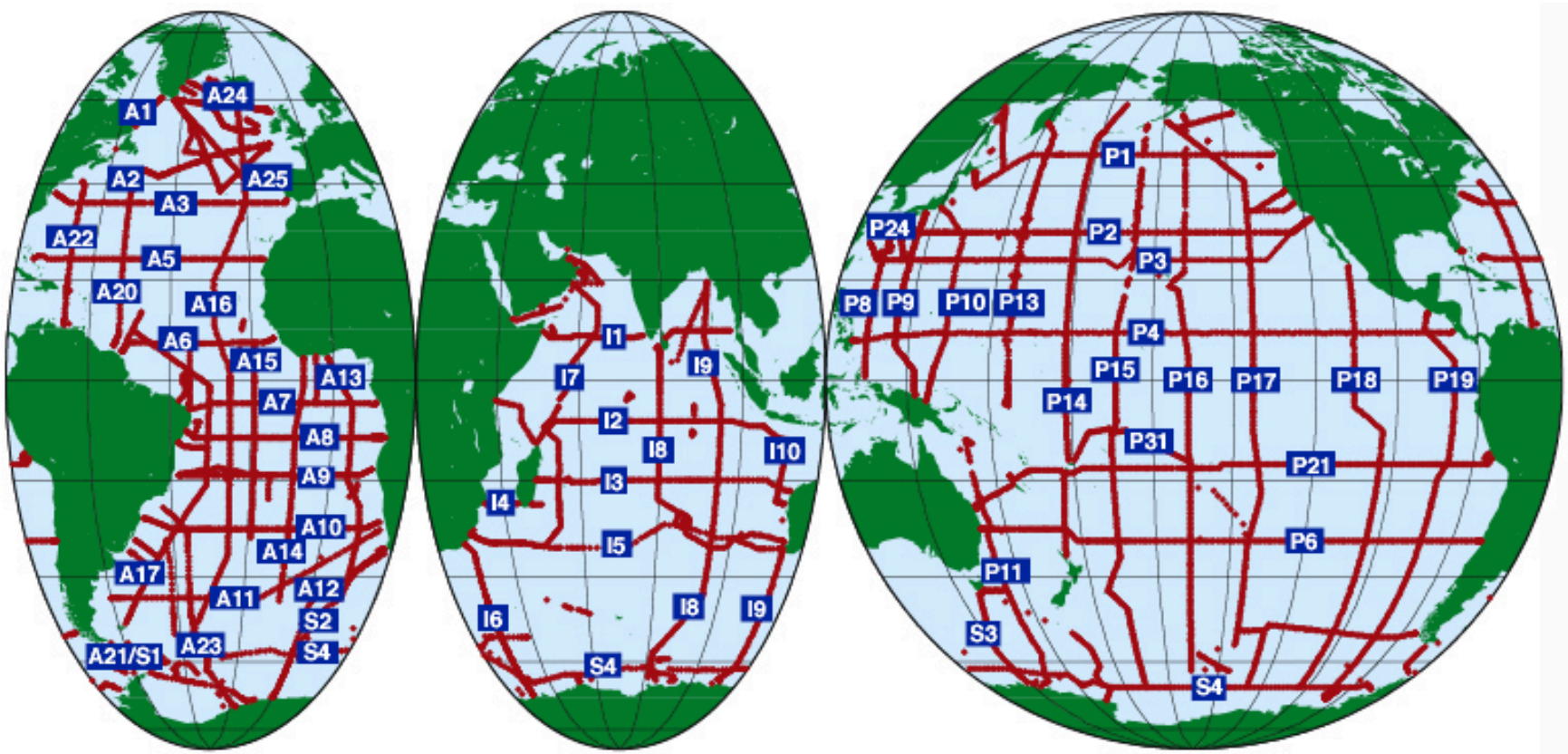


An EXpendable BathyThermograph (XBT) with it's probe inside a canister (top). On the right is an exploded view of the XBT.

http://www.aoml.noaa.gov/phod/soop/index.php

Ocean Observations

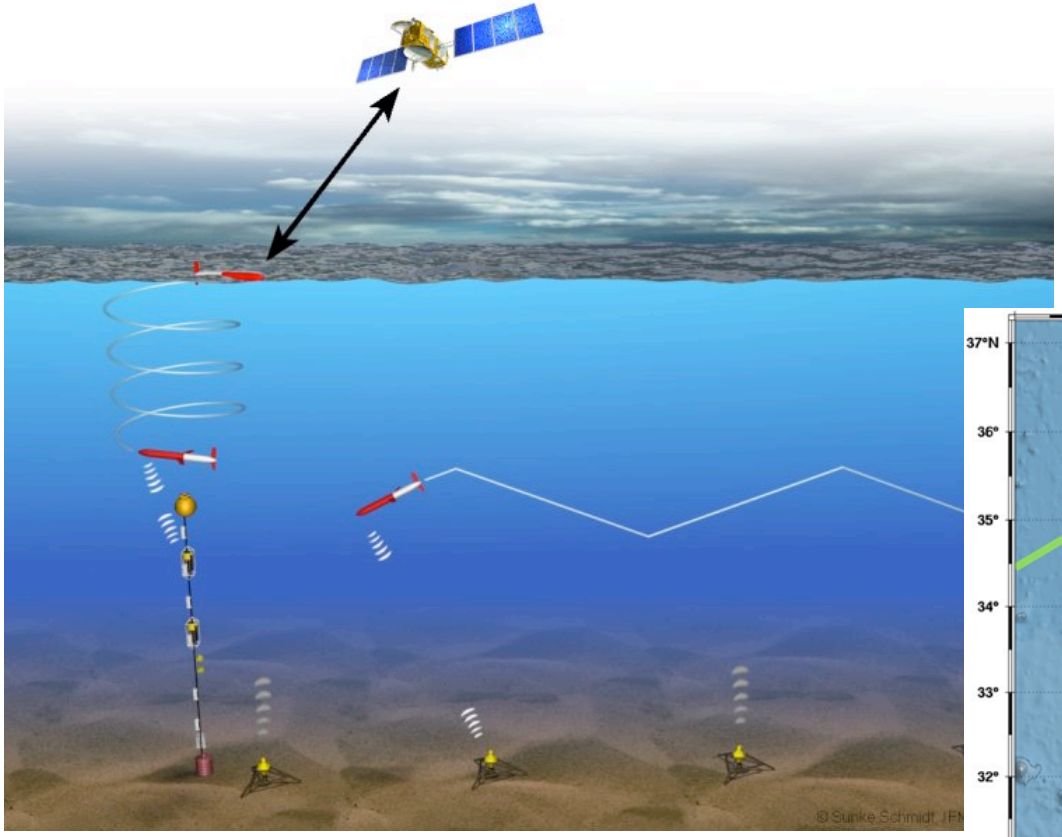
- In situ: Research Vessels (over 10yrs) by WOCE

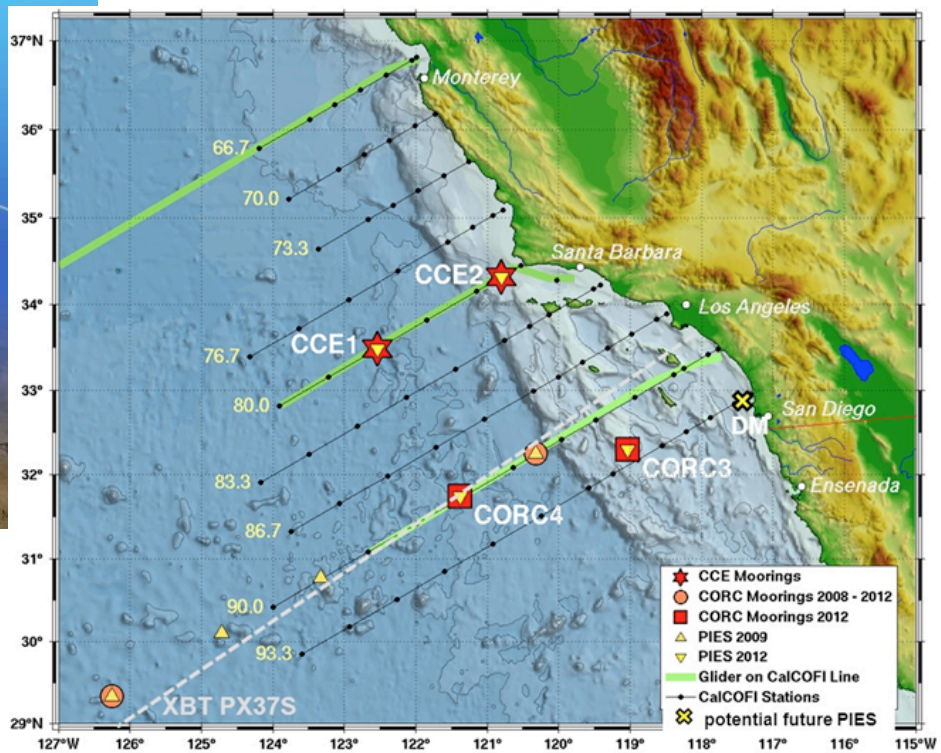


http://woceatlas.tamu.edu/printed/SOA_WOCE.html

Ocean Observations

- In Situ: Ocean Gliders (limited to targeted field campaign)

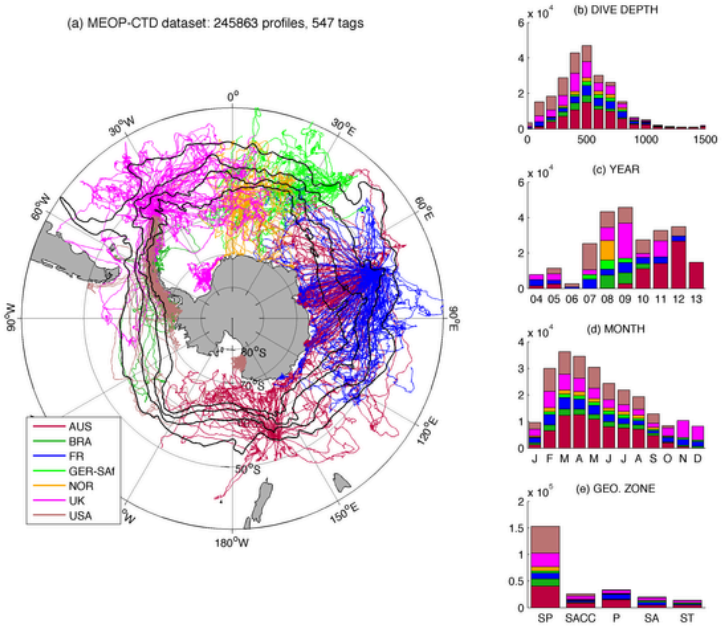




http://mooring.ucsd.edu/index.html?/projects/corc/corc_intro.html

Ocean Observations

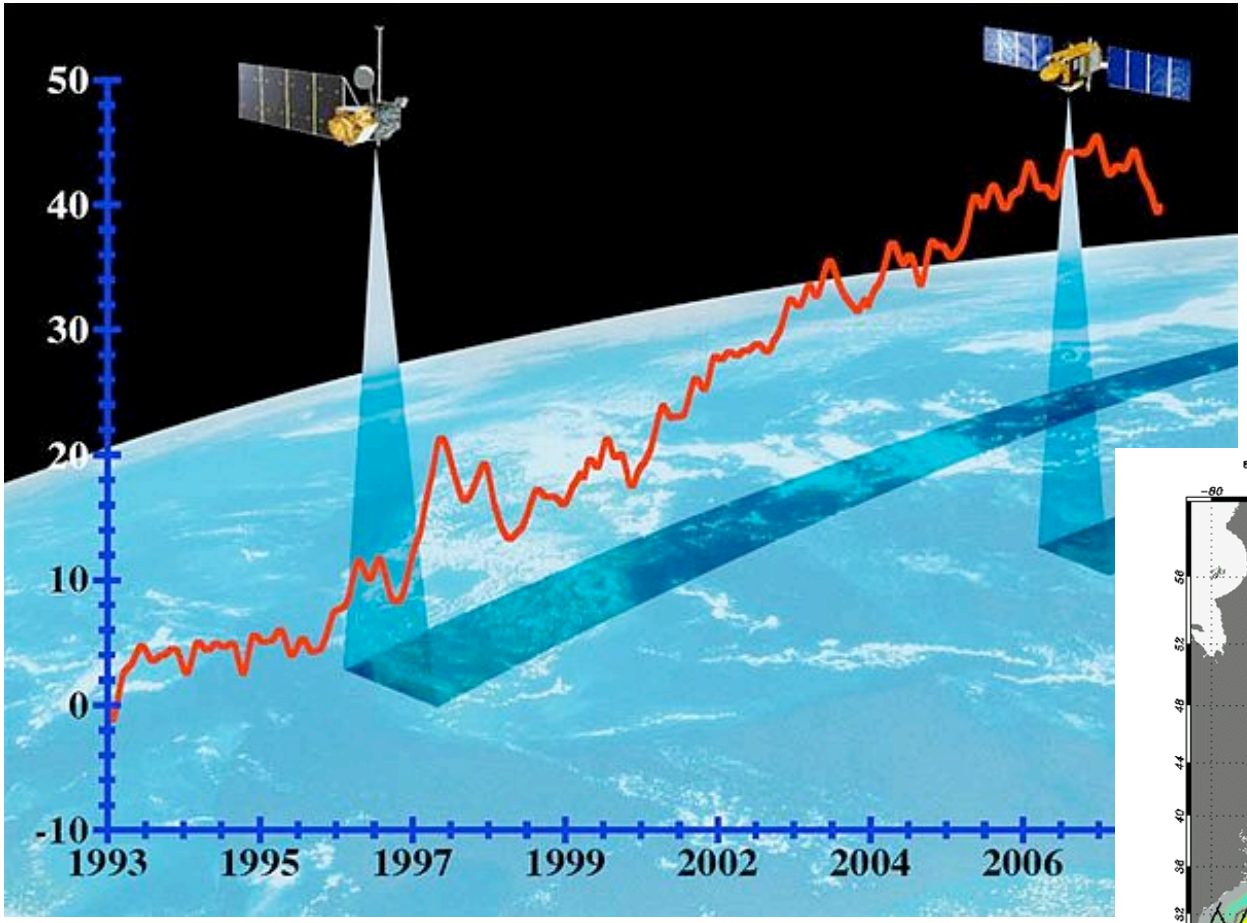
- In Situ: Marine Mammals

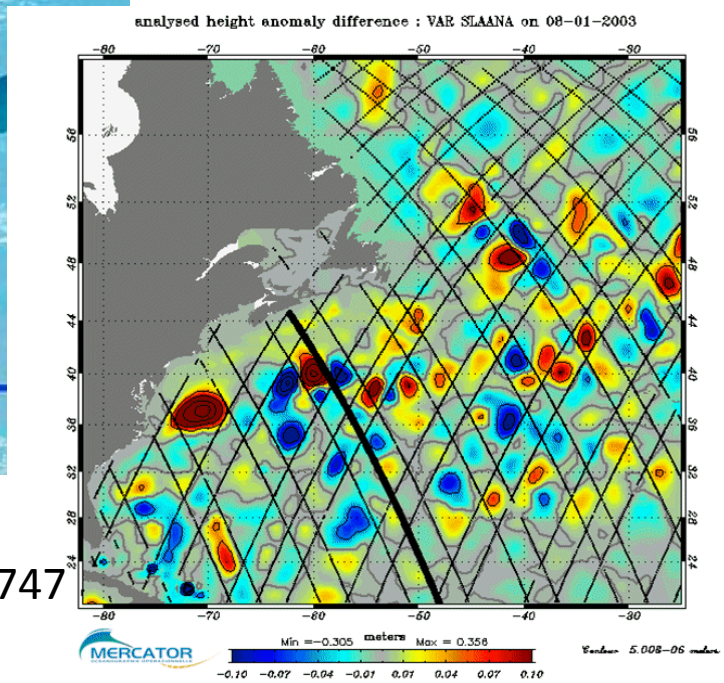


https://www.godae-oceanview.org/ & UKMO

Ocean Observations

- Satellite: Altimetry





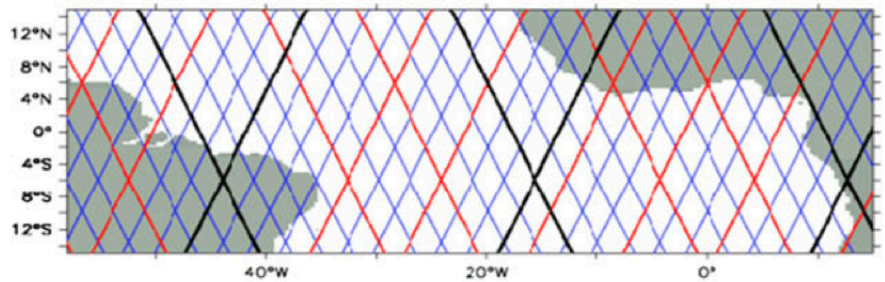
http://www.jpl.nasa.gov/news/features.cfm?feature=1747

Ocean Observations

- Satellite: Altimetry

Repeat Period

10days

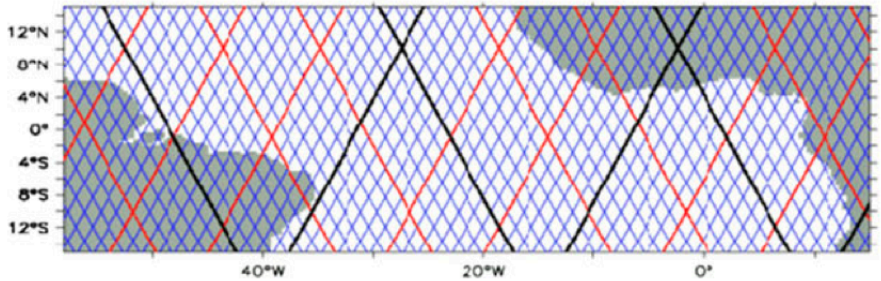


Orbit-1 (Jason)

H=1336km i=66°

(sub-)cycles (days) : **0.9** **3.3** **9.9**

17days

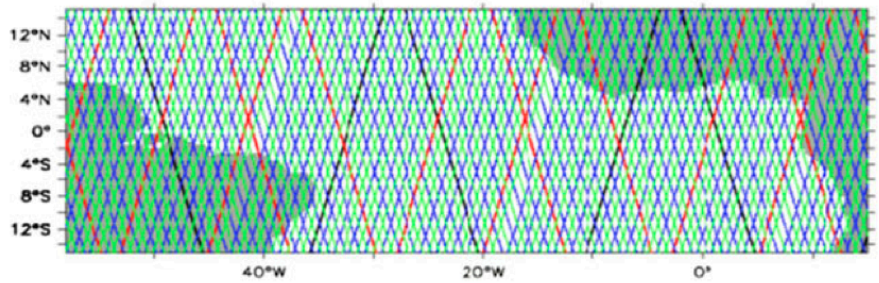


Orbit-2 (Gfo) : H=800km i=108°

H=800km i=108°

(sub-)cycles (days) : **1.0** **2.8** **17.0**

35days



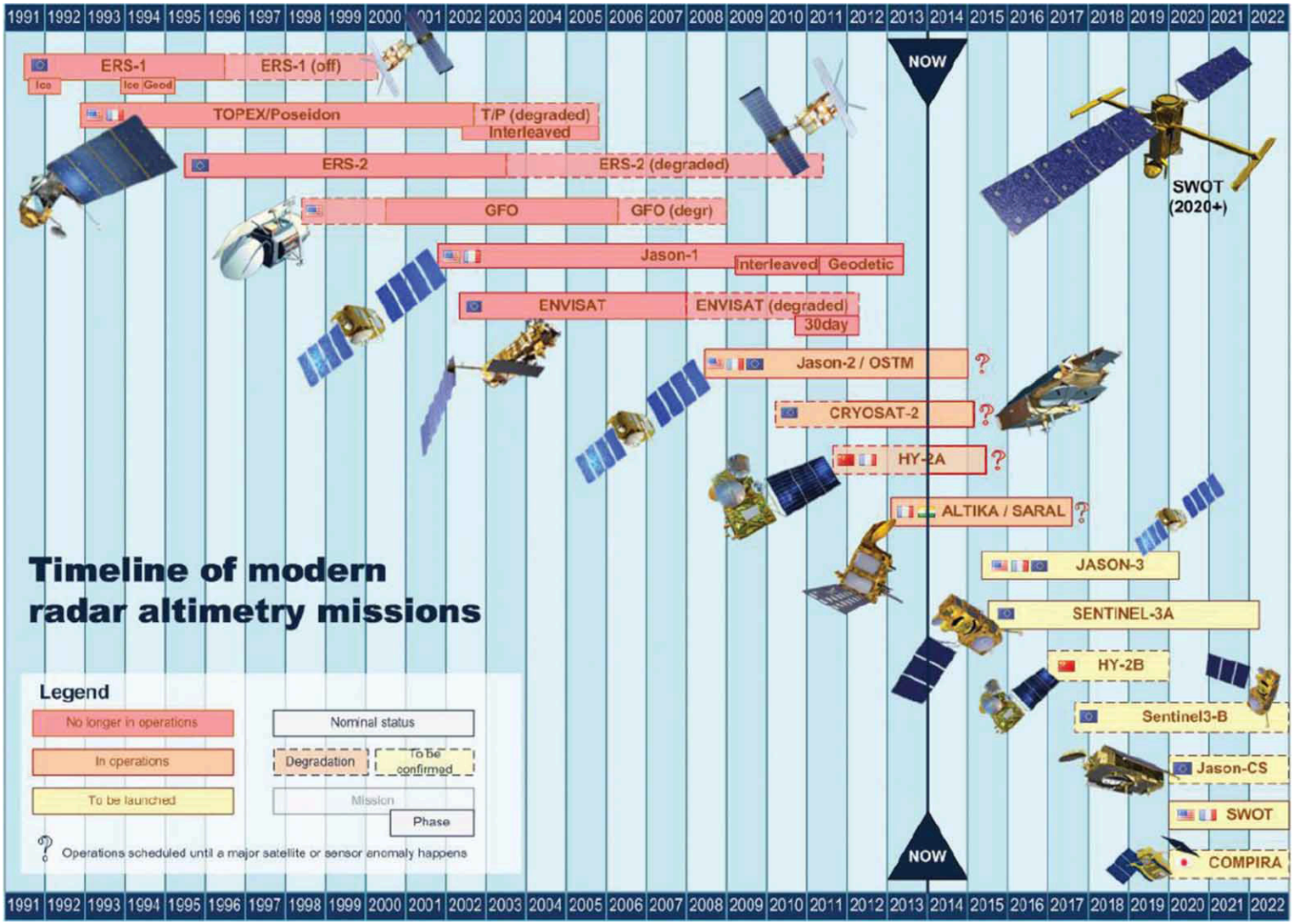
Orbit-3 (Envisat, Saral

H=782km i=98°

(sub-)cycles (days) : **1.0** **3.0** **17.5** **35.0**

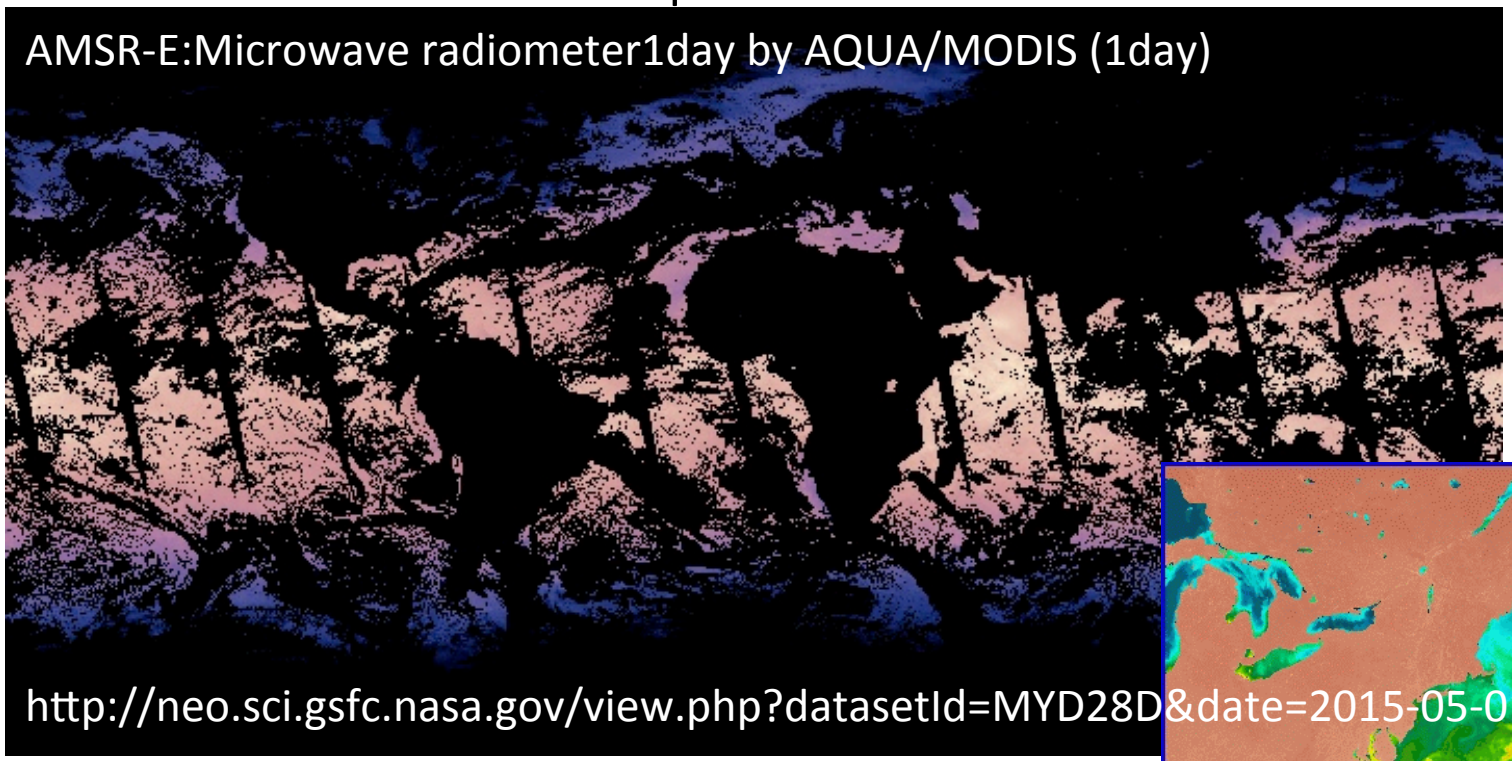
Ocean Observations

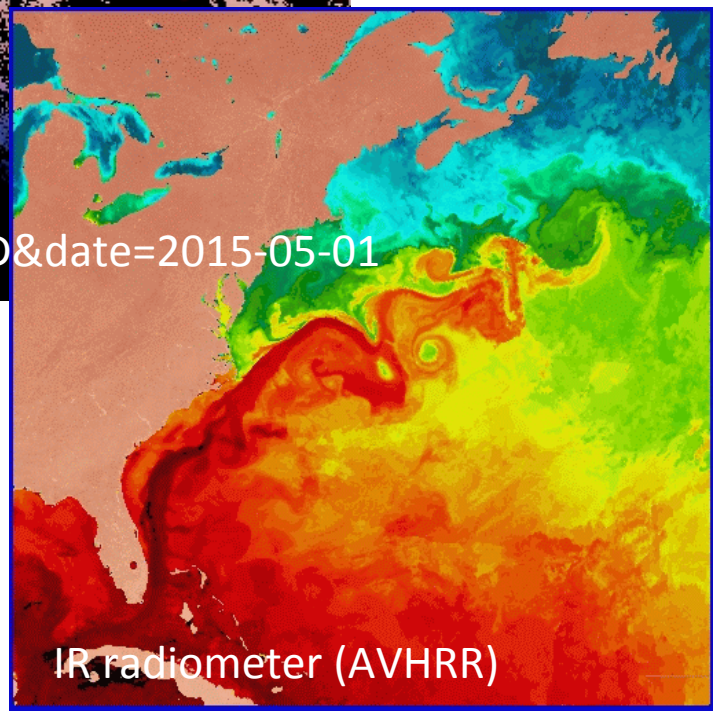
- Satellite: Altimetry



Ocean Observations

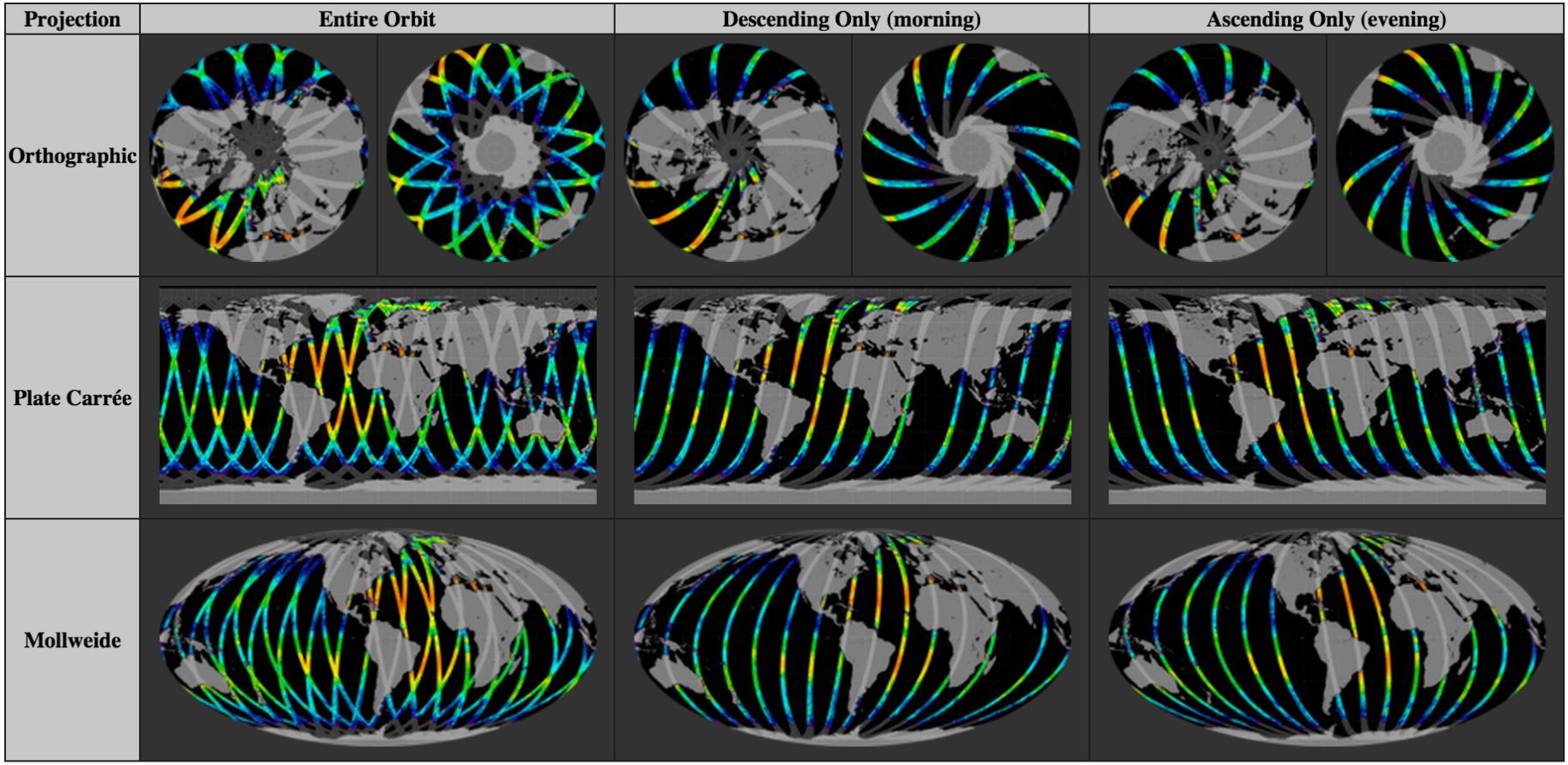
- Satellite: Sea Surface Temperature





Ocean Observations

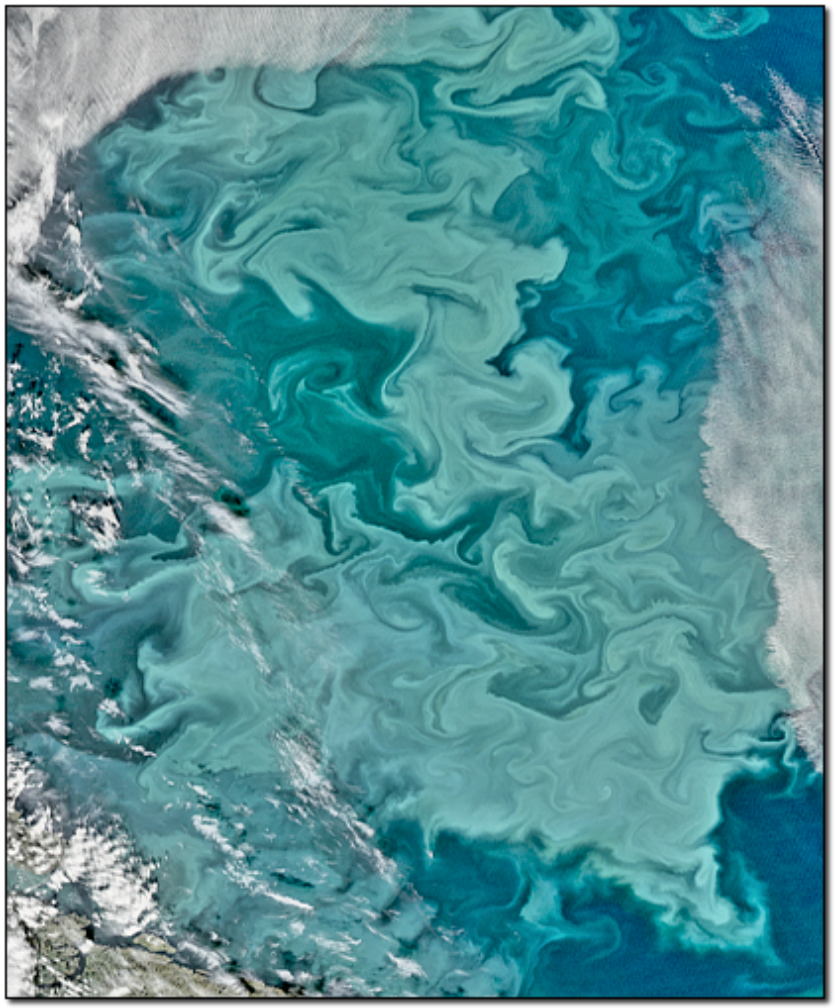
- Satellite: Sea Surface Salinity (daily 31 May 2015)



http://aquarius.umaine.edu/

Ocean Observations

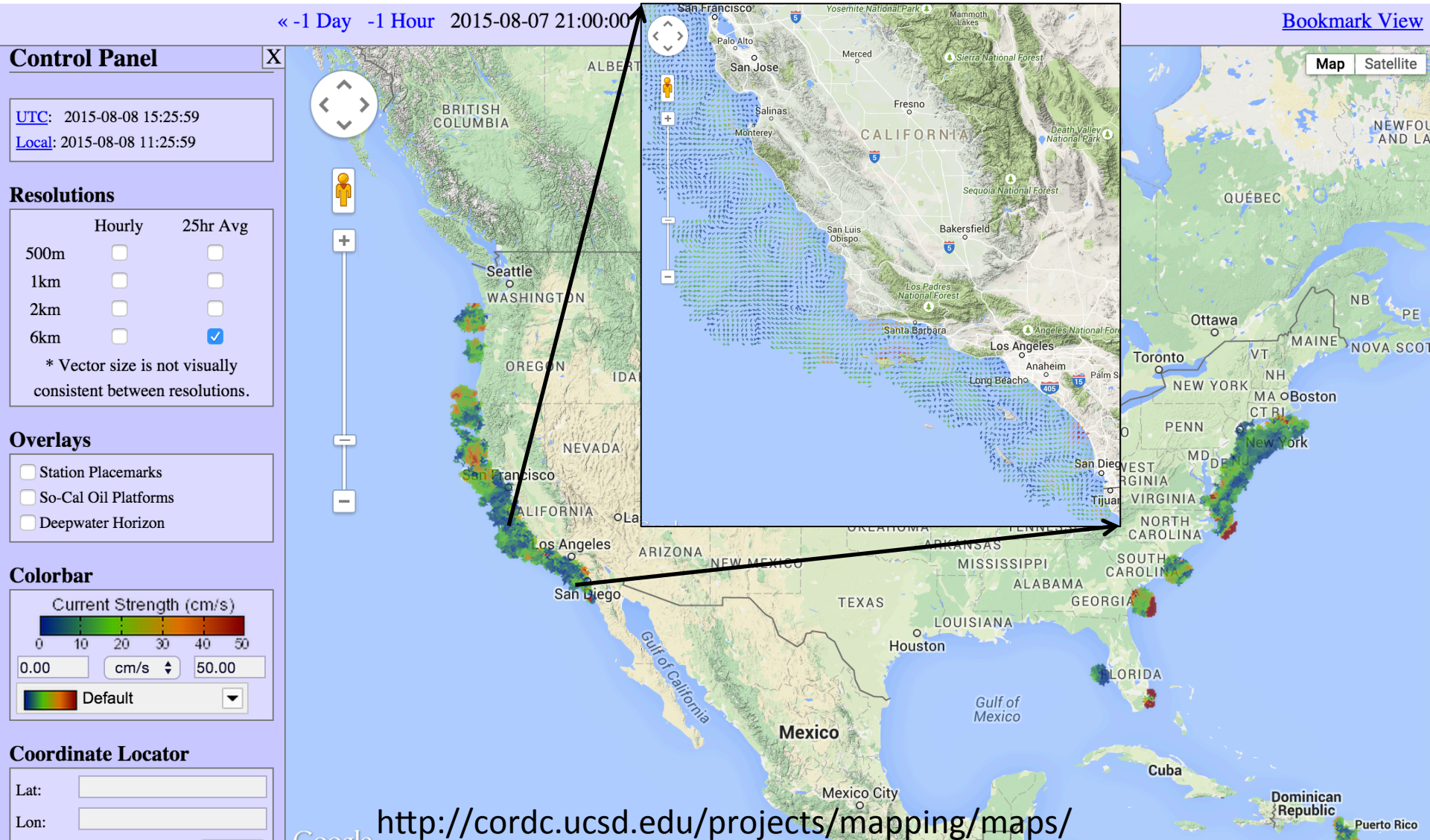
- Satellite: Ocean Color Sensors (chlorophyll) for primary production



http://oceancolor.gsfc.nasa.gov/cms/homepage

Ocean Observations

- Remote Sensing: High Frequency Radar for Ocean Current



http://cordc.ucsd.edu/projects/mapping/maps/

Ocean Observations

- In situ
  - Diverse resources
  - Sparse and sporadic
- Satellite
  - Large amount (relatively speaking to in situ)
  - Observation of sea surface

Future satellite oceanography observing challenges can be

(1) continuity and reliability

(2) resolution and coverage

(3) knowledge

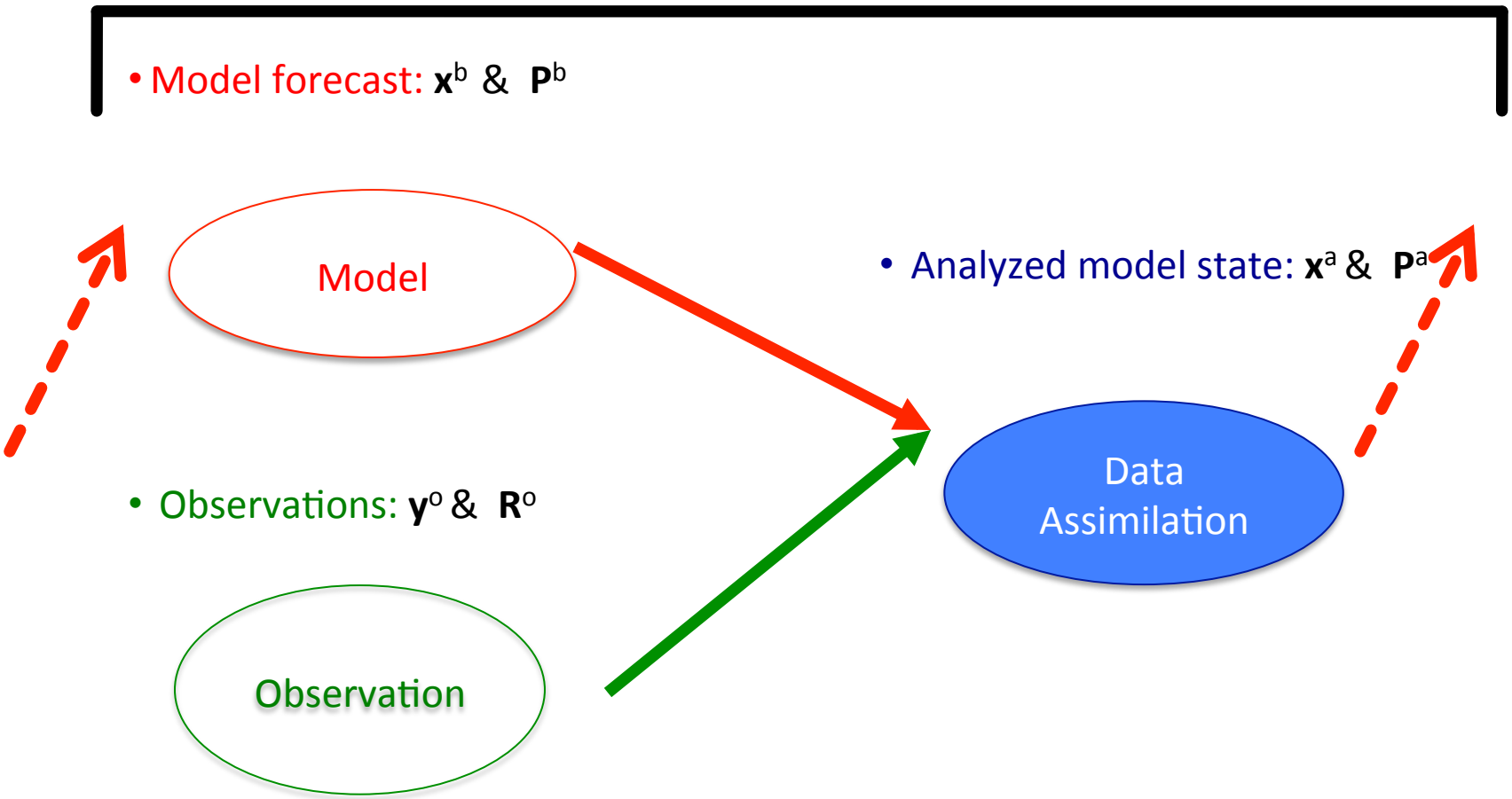
[Le Traon, et al 2015]

For both in situ or satellite

Uncertainty (instrumental, representativeness) are difficult to estimate

Schematic of Data Assimilation: Assimilation Window

- Data assimilation is a iterative method that iterates the cycle over a window:



Ocean Data Assimilation

- For OO, mostly 3DVar/OI
- GODAE Forecast Systems

System	Assimilation Scheme	SST	SSH	Other
BLUElink>	BODASv8.3 BODAS	WindSAT, NAVO (AVHRR), NOAA-18, MetopA,, NOAA-19 AMSR-E, Pathfinder, NAVO	Jason2, Cryosat-2, SARAL/AltiKa IGDR data streams Along-track altimetry (all altimeters)	Adaptive Non-linear Dynamical Initialisation (T,S,u,v,η)
CONCEPTS	SAM2 (Mercator-PSY2V3R1) - ice 3Dvar	Canadian Meteorological Centre SST Analysis	AVISO	Coriolis/CLS (in situ T&S) as well as sea ice data from satellite sources (AMSR, AVHRR, MODIS, Radarsat) and ship reports.
ECCO group	Adjoint, Kalman filter/smoother, Green's function	AMSR-E, TMI, AVHRR	Along-track TOPEX/Poseidon, Jason1 & -2, ENVISAT, GFO, ERS-1 & -2, tide gauges	XBT, CTD, Argo, TOGA-TAO, SSM/I (sea ice), GRACE, scatterometers, sea surface salinity, Florida Current transport
ECMWF	NEMOVAR	NOAA OI v2 1x1 weekly, and OSTIA since 2010	AVISO along track sea level anomalies and global maps	XTB,CTD,Argo,Moorings from the EN3 (XBT corrected) data set until 2010 and from the GTS thereafter
FOAM	NEMOVAR (3DVar-FGAT)	GHRSST data including AVHRR from NOAA and MetOp. Also in situ surface data from ships, moored buoys and drifting buoys	Along-track satellite altimeter data from Aviso, including Jason-2 and Cryosat-2 data	All available in situ temperature and salinity data including XBTs, Argo, CTDs, moored buoys, gliders, marine mammals; SSM/I sea-ice concentration data from EUMETSAT OSI-SAF
HYCOM/NCODA	Navy Coupled Ocean Data Assimilation (NCODA): 3DVAR	NOAA-18, NOAA-19, METOP-A, METOP-B, GOES-13,	Jason-2, Altika, Cryosat-2	In situ profiles, (Argo, gliders, XBTs, fixed and drifting buoys, CTDs), sea ice concentration (SSM/I, SSMIS).

https://www.godae-oceanview.org

Ocean Data Assimilation

- For OO, mostly 3DVar/OI
- GODAE Forecast Systems

INDOFOS	3D-VAR (MOM-GODAS), Ensemble OI (HyCOM), OI (ROMS, experimental)	Relaxed to Reynolds SST (GODAS)	AVISO (HyCOM)	T,S profiles (ARGO, XBT, XCTD, moored buoys) in GODAS
Mercator Ocean	"Mercator Assimilation System" version 2 (SAM-2), reduced-order Kalman filter based on the SEEK formulation with a 3DVAR bias correction for temperature and salinity	Reynolds AVHRR 0.25° SST	SSALTO/DUACS along track SLA (AVISO), Mean Dynamic Topography constructed with CNES_CLS09 MDT and GLORYS reanalysis	T/S profiles from Coriolis, Sea ice concentration from CERSAT
MFS	3DVAR (OCEANVAR)	L4 daily maps Only nudging (variational assimilation)	L3 Along track sub-sampled data (every second point)	XBTs, Argo, CTDs. GLIDER are pre-operational; L3 Chl maps (5 days mean)
MOVE/MRI.COM	3DVAR (Global, North Pacific), 4DVAR (Western North Pacific, Japan Area, Seto-Inland Sea) with EOF decomposition, quasi-Newton methods, FGAT, and variational dynamic QC	MGDSST (JMA-GHRSST product)	Jason-2 (Cryosat-2 and SARAL will be added)	Argo, GTS
NMEFC	3DVAR EnOI OVALS	RTGSST MGDSST Reynolds SST	Along-track sea level anomaly from CLS, including Jason-1, Jason-2 and Cryosat-2 data. T/P	T&S Argo profiles, XBT
REMO	OI with fixed model covariance	HYCOM + NCODA system	HYCOM+NCODA system	None
RTOFS	NRL / NAVOCEANO NCODA (See USA-HYCOM for details)	See USA-HYCOM	See USA-HYCOM	See USA-HYCOM
TOPAZ	DEnKF (Sakov and Oke 200*)	OSTIA	Along track SLA from CLS	SSM/I (ice-concentration) from EUMETSAT, T&S profiles from MyOcean Arctic in-situ TAC (incl Argo, CTD and ITPs)

GODAE: Global Ocean Data Assimilation Experiments

Mission:

- Define, monitor and promote actions aimed at coordinating and integrating research associated with multi-scale and multi-disciplinary ocean analysis and forecasting systems

International Coordination:

- Consolidation and improvement of global/regional analysis and forecasting systems
- The progressive development and scientific testing for the next generation
  - Biogeochemical and ecosystems
  - Extending from the open ocean into shelf seas and coastal waters
- The exploitation of the capability in other applications

Current Status of OO

- Need of assessment for ocean observing system:
  - contribution of the various scientific guidance
  - improved design and implementation

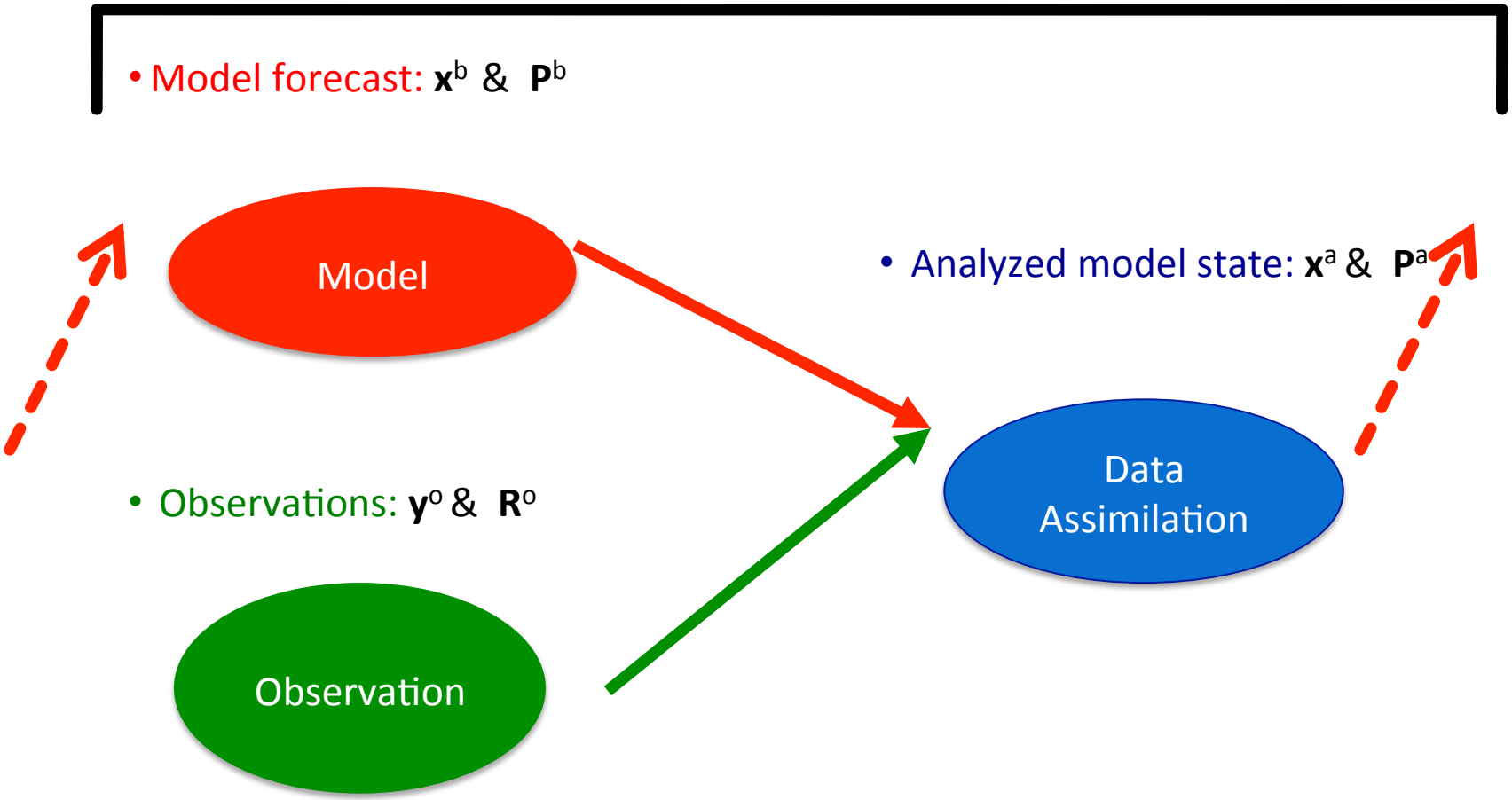
- Need of transition

  from a demonstration

  to a permanent and sustained capability

Schematic of Data Assimilation: Assimilation Window

- Data assimilation is a iterative method that iterates the cycle over a window:



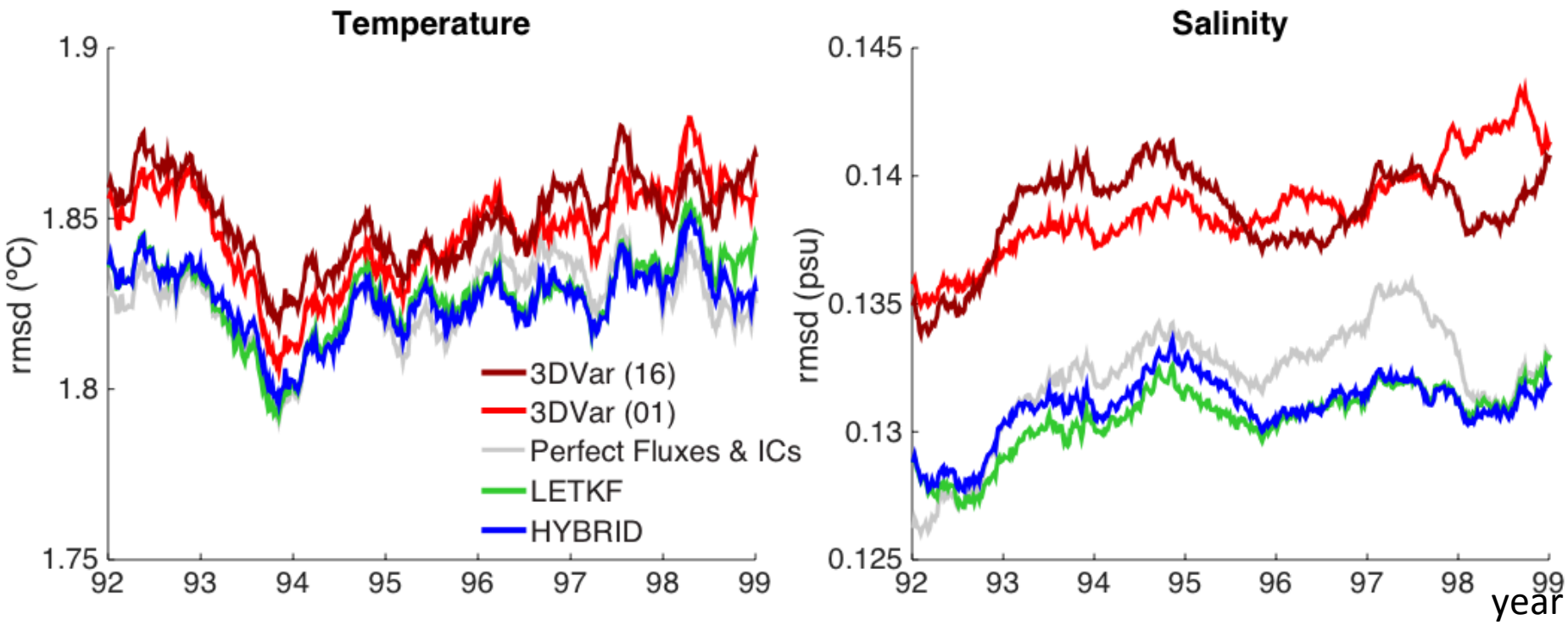
Lots of room & serious needs for improvement

Demonstration: Global Data Assimilation

- Application of hybrid-gain data assimilation [Penny et al , 2015] To NOAA GODAS
  - Model:MOM4p1(1/2x1/4º)
  - Data Assimilation
    - 3DVar : $\Delta\mathbf{x}^a_{3D}$ (current operational system)
      - » Analysis variables: Temperature, Salinity, U/V velocities
    - LETKF: $\Delta\mathbf{x}^a_{en}$
      - » Localization is applied in the horizontal with 700km
      - » No localization is applied in the vertical
    - Hybrid-Gain method
      - » $\Delta\mathbf{x}^a = (1-\alpha)\Delta\mathbf{x}^a_{3D} + \alpha\Delta\mathbf{x}^a_{en}$

Hybrid-Gain OSSE to GODAS

- OSSE Experiments

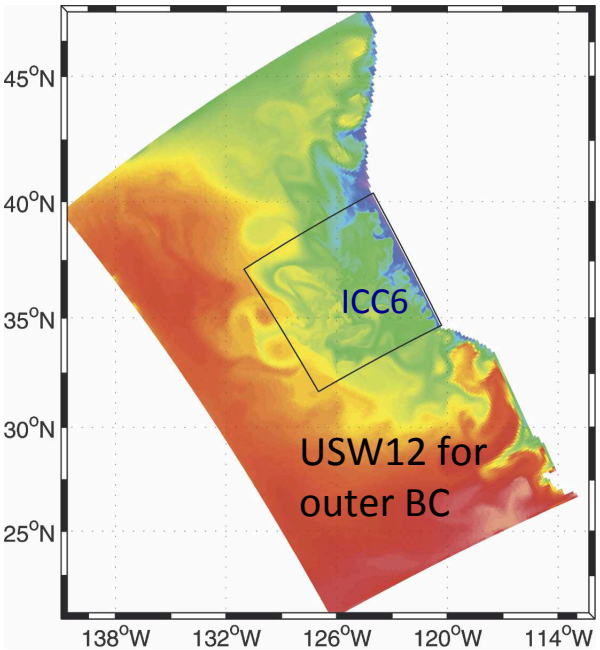


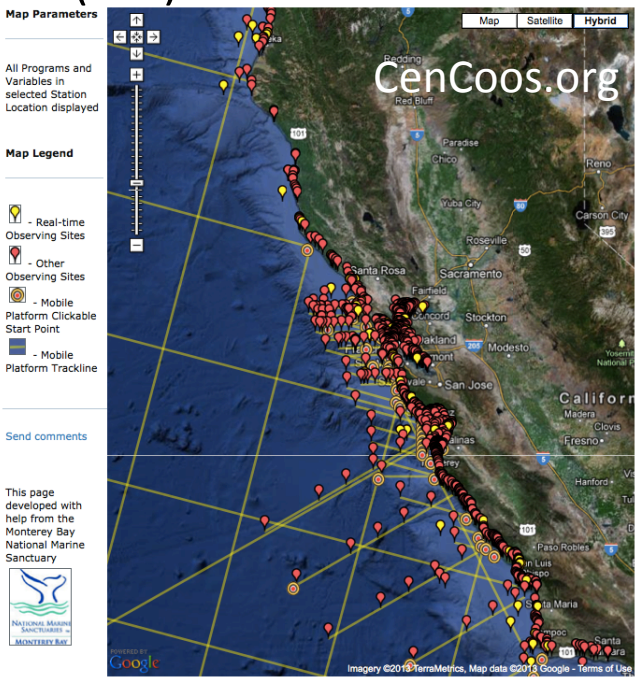
- Ensemble approaches reduced RMSE of forecast and analysis errors vs. 3DVar
- The Hybrid reduced biases imposed on LETKF via surface forcing conditions where observations are sparse.
- Real obs experiments are on-going

Courtesy of S. Penny

Motivation

- Original: Multi-Scale (MS) & Multi-Resolution (MR) in Coastal Ocean data





[Li et al, 2015ab]

- Many data assimilation systems exhibit MS/MR

Motivation: Dynamics and Model

- Scale and Resolution
  - Geophysical fluid dynamics: Multi-Scale (MS) phenomena
  - Computational modeling & data assimilation: Multi-Resolution (MR)

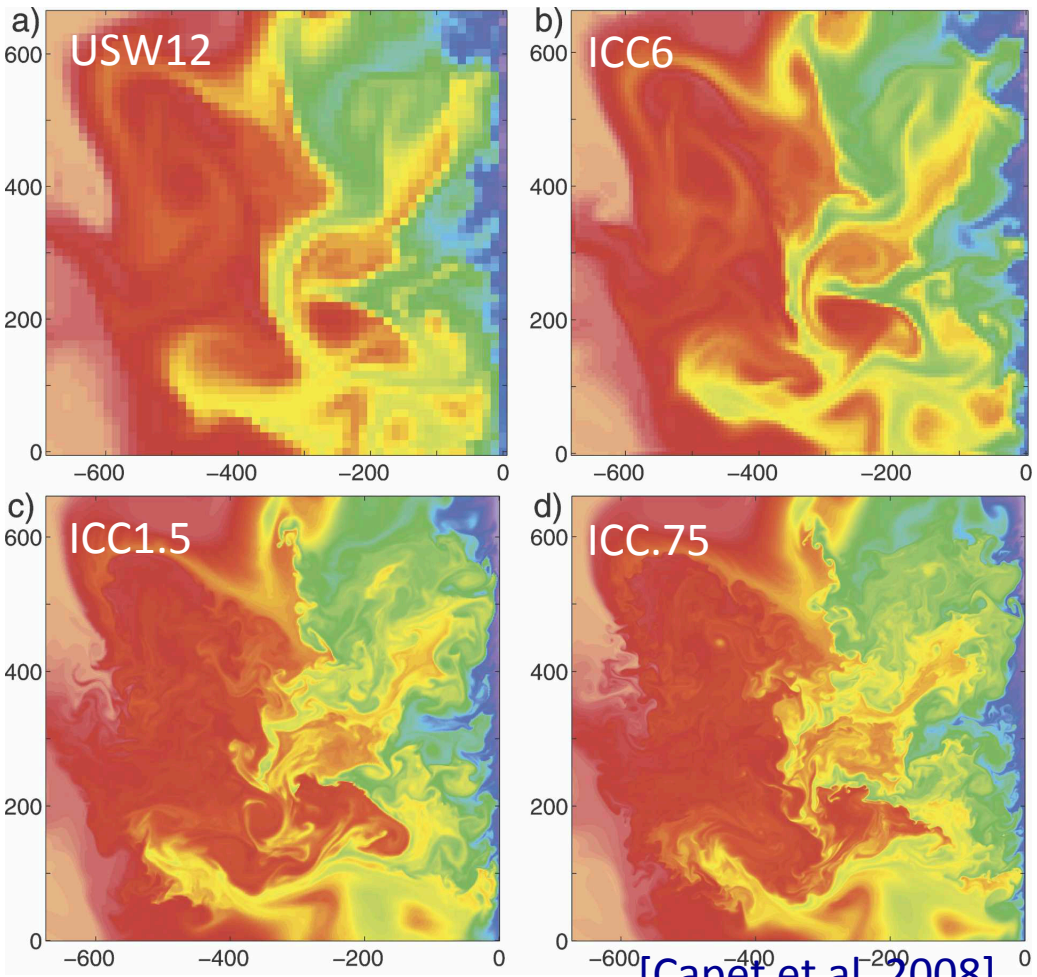
- Model state **x**: spatial MS-MR

$$\mathbf{x} = \mathbf{x}_L + \mathbf{x}_S \quad [+\ldots]$$

$\mathbf{x}_L$: large-scale

$\mathbf{x}_S$: smaller-scale

  - High-Res model
    - $\mathbf{x}_L$ and $\mathbf{x}_S$: both present
  - Low-Res model
    - $\mathbf{x}_L$: may be OK
    - $\mathbf{x}_S$: may not be so OK



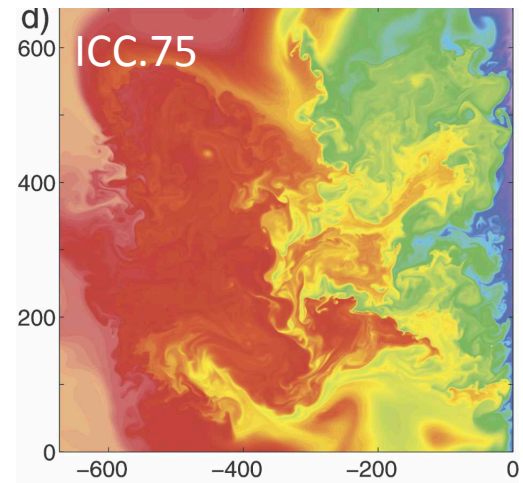
[Capet et al, 2008]

Analysis Increment: Assimilation of Data

- Standard VAR formulation
  - Cost function for analysis increment $\Delta\mathbf{x}=\mathbf{x}-\mathbf{x}^b$

$$J(\Delta\mathbf{x})=\frac{1}{2}\Delta\mathbf{x}^T(\mathbf{P}^b)^{-1}\Delta\mathbf{x}$$

$$+\frac{1}{2}(\mathbf{y}^o-\mathbf{h}(\mathbf{x}^b+\Delta\mathbf{x}))^T(\mathbf{R}^o)^{-1}(\mathbf{y}^o-\mathbf{h}(\mathbf{x}^b+\Delta\mathbf{x}))$$



$\mathbf{x}^b,\mathbf{P}^b$: Background state and error covariance

$\mathbf{y}^o,\mathbf{R}^o$: Observation and its error covariance

$\mathbf{h}, \mathbf{H}$: Observation operator and its tangent linear model

- Analysis increment by the optimization is the same as the Minimum Variance (MV) estimation for the quadratic cost function

$$\Delta\mathbf{x}^a = \underline{\mathbf{P}^b}\mathbf{H}^T(\mathbf{H}\mathbf{P}^b\mathbf{H}^T+\mathbf{R}^o)^{-1}\underline{(\mathbf{y}^o-\mathbf{h}(\mathbf{x}^b))}$$

Innovation **d** (difference between obs & bg In obs space)

Regression to model space

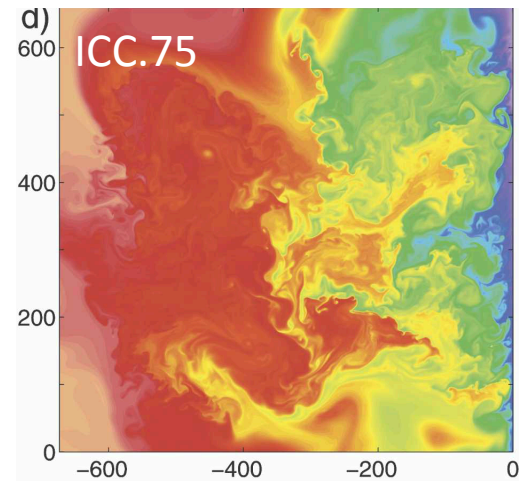
Bg covariance in obs space

Obs error covariance

Analysis Increment: Assimilation of Data

- MSDA scheme development

$$J(\Delta\mathbf{x}) = \frac{1}{2}\Delta\mathbf{x}^T(\mathbf{P}^b_{LL} + \mathbf{P}^b_{SS} + \mathbf{P}^b_{LS} + \mathbf{P}^b_{LS})^{-1}\Delta\mathbf{x}$$
$$+\frac{1}{2}(\mathbf{d}-\mathbf{H}\Delta\mathbf{x})^T\mathbf{R}^{-1}(\mathbf{d}-\mathbf{H}\Delta\mathbf{x})$$



- Using 2 auto-scale bg covariances $\mathbf{P}^b_{LL}$ & $\mathbf{P}^b_{SS}$
  ← Straightforward to include cross-scale bg covariances $\mathbf{P}^b_{LS}$ & $\mathbf{P}^b_{SL}$

- Based on VAR formulation
  ←Straightforward to reformat as EnSRF or LETKF

- Based on incremental VAR formulation
  ←Straightforward to reformat as full VAR

MS-MR Formulation for Background

- MS-MR Variation for $\Delta\mathbf{x}^a = \Delta\mathbf{x}^a{}_L + \Delta\mathbf{x}^a{}_S$

Scheme 1. Simultaneous decomposition for multi-resolution (MR)

$$J_L(\Delta\mathbf{x}_L) = \frac{1}{2}(\Delta\mathbf{x}_L)^T(\mathbf{P}_L^b)^{-1}\Delta\mathbf{x}_L + \frac{1}{2}(\mathbf{d}-\mathbf{H}\Delta\mathbf{x}_L)^T(\mathbf{R}^o + \mathbf{H}\mathbf{P}_S^b\mathbf{H}^T)^{-1}(\mathbf{d}-\mathbf{H}\Delta\mathbf{x}_L)$$

Representativeness error

$$J_S(\Delta\mathbf{x}_S) = \frac{1}{2}(\Delta\mathbf{x}_S)^T(\mathbf{P}_S^b)^{-1}\Delta\mathbf{x}_S + \frac{1}{2}(\mathbf{d}-\mathbf{H}\Delta\mathbf{x}_S)^T(\mathbf{R}^o + \mathbf{H}\mathbf{P}_L^b\mathbf{H}^T)^{-1}(\mathbf{d}-\mathbf{H}\Delta\mathbf{x}_S)$$

- individually takes background more seriously than original
- increment is addition over the scales

Note

» Same solution for solving $J(\Delta\mathbf{x})$ with $\mathbf{P}^b = \mathbf{P}^b{}_L + \mathbf{P}^b{}_S$
but computationally efficient

✦ $\Delta\mathbf{x}^a{}_L$ at low resolution with $\mathbf{P}^b{}_L$

✦ $\Delta\mathbf{x}^a{}_S$ at high resolution with $\mathbf{P}^b{}_S$

MS-MR Formulation for Background

- MS-MR Variation for $\Delta\mathbf{x}^a = \Delta\mathbf{x}^a{}_L + \Delta\mathbf{x}^a{}_S$

Scheme 1. Simultaneous decomposition for multi-resolution (MR)

$$J_L(\Delta\mathbf{x}_L) = \frac{1}{2}(\Delta\mathbf{x}_L)^T(\mathbf{P}_L^b)^{-1}\Delta\mathbf{x}_L + \frac{1}{2}(\mathbf{d}-\mathbf{H}\Delta\mathbf{x}_L)^T(\mathbf{R}+\mathbf{H}\mathbf{P}_S^b\mathbf{H}^T)^{-1}(\mathbf{d}-\mathbf{H}\Delta\mathbf{x}_L)$$

$$J_S(\Delta\mathbf{x}_S) = \frac{1}{2}(\Delta\mathbf{x}_S)^T(\mathbf{P}_S^b)^{-1}\Delta\mathbf{x}_S + \frac{1}{2}(\mathbf{d}-\mathbf{H}\Delta\mathbf{x}_S)^T(\mathbf{R}+\mathbf{H}\mathbf{P}_L^b\mathbf{H}^T)^{-1}(\mathbf{d}-\mathbf{H}\Delta\mathbf{x}_S)$$

Note

» $\Delta\mathbf{x}_L$ is usually dominant

→Solving for $\Delta\mathbf{x}_L$ first gives better initialization for $\Delta\mathbf{x}_S$

by improving innovation $\mathbf{d} = \mathbf{y}^o - \mathbf{H}(\mathbf{x}^b{}_L + \mathbf{x}^b{}_S)$

to $\mathbf{d}^* = \mathbf{y}^o - \mathbf{H}(\mathbf{x}^b{}_L + \Delta\mathbf{x}^a{}_L + \mathbf{x}^b{}_S)$

MS-MR Formulation for Background

- MS-MR Variation for $\Delta\mathbf{x}^a = \Delta\mathbf{x}^a{}_L + \Delta\mathbf{x}^a{}_S$

Scheme 2. Successive Approach **[Li et al, 2015a,b]**

1. Large scale (unchanged)

$$J_L(\Delta\mathbf{x}_L) = \frac{1}{2}(\Delta\mathbf{x}_L)^T(\mathbf{P}_L^b)^{-1}\Delta\mathbf{x}_L + \frac{1}{2}(\mathbf{d}-\mathbf{H}\Delta\mathbf{x}_L)^T(\mathbf{R}+\mathbf{H}\mathbf{P}_S^b\mathbf{H}^T)^{-1}(\mathbf{d}-\mathbf{H}\Delta\mathbf{x}_L)$$

2. Small scale, successively after large scale using $\Delta\mathbf{x}^a{}_L$

$$J_S(\Delta\mathbf{x}_S) = \frac{1}{2}(\Delta\mathbf{x}_S)^T(\mathbf{P}_S^b)^{-1}\Delta\mathbf{x}_S + \frac{1}{2}(\mathbf{d}_S^* - \mathbf{H}\Delta\mathbf{x}_S)^T(\mathbf{R}+\mathbf{H}\mathbf{P}_L^a\mathbf{H}^T)^{-1}(\mathbf{d}_S^* - \mathbf{H}\Delta\mathbf{x}_S)$$

Using more "accurate" observations

updated innovation: $\mathbf{d}^*{}_S = \mathbf{d} - \mathbf{H}\Delta\mathbf{x}^a{}_L$

updated obs error covariance: $\mathbf{R} + \mathbf{H}\mathbf{P}^a{}_L\mathbf{H}^T$

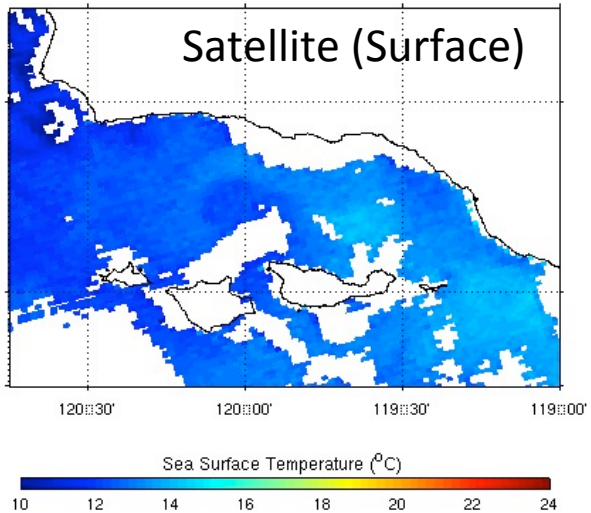
Representativeness error

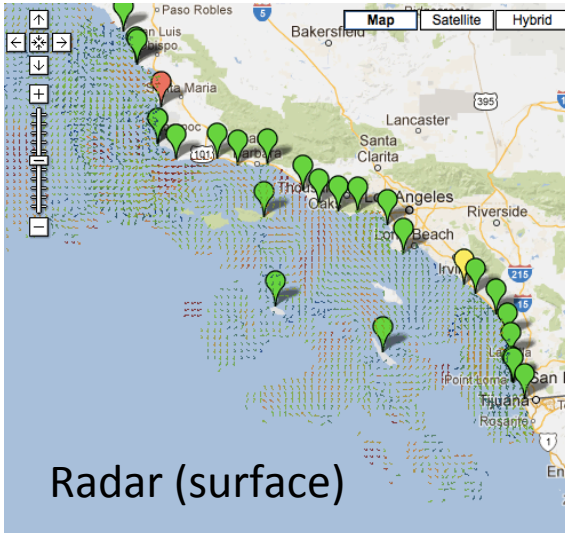
Note:

» Similar idea to Successive Covariance Localization (Zhang et al, 2009)

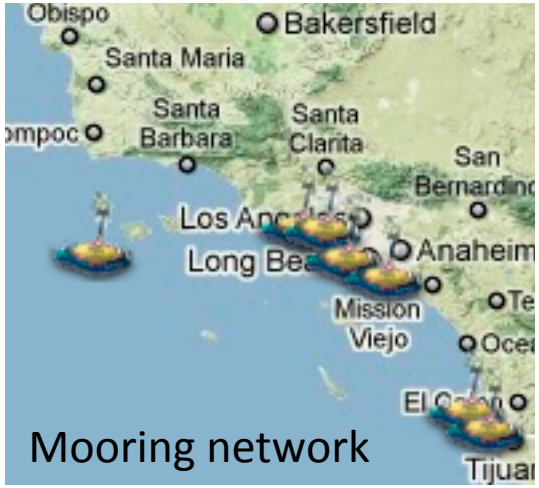
Coastal Ocean Observation Network

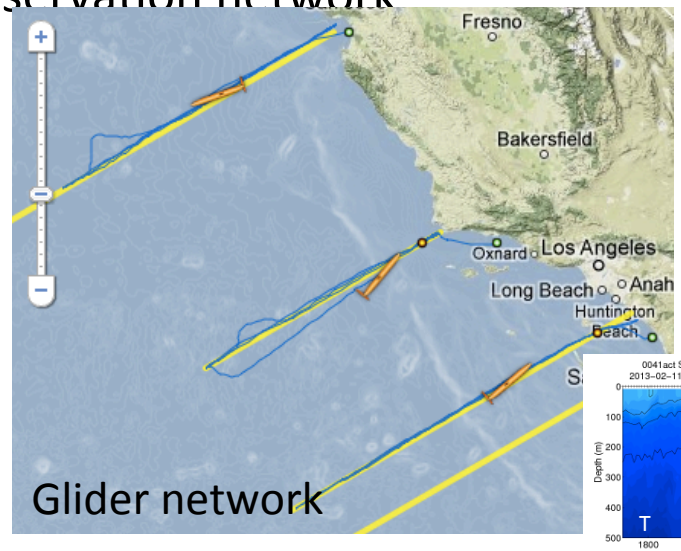
- $\mathbf{y}_D=\mathbf{H}_D\mathbf{x}$: Dense observation network

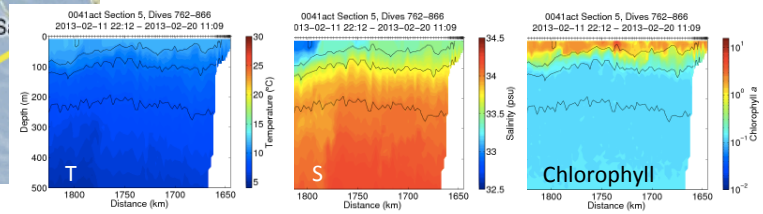




- $\mathbf{y}_c=\mathbf{H}_c\mathbf{x}$: Coarse observation network





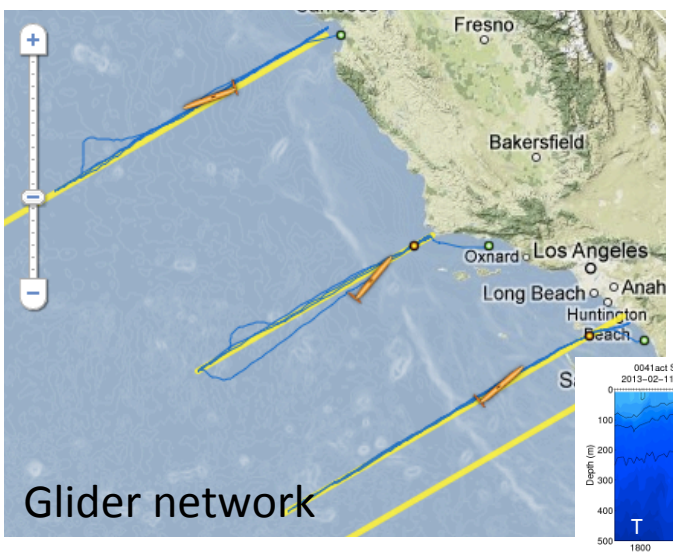


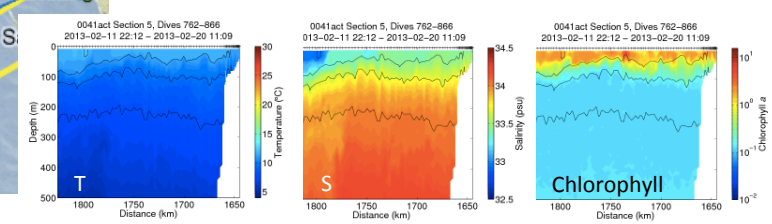
Motivation: Multi-Resolution (MR) Observing Network

- Spatial density of observing system varies from one obs type to another
  - Coarse network:
    - Mooring
    - Gliders

$$\mathbf{y}_c = \mathbf{H}_c \mathbf{x} \qquad \text{with } \mathbf{R}_c$$

→ MS schemes will lead to MS increment through MS $\mathbf{P}^b = \mathbf{P}^b_L + \mathbf{P}^b_S$



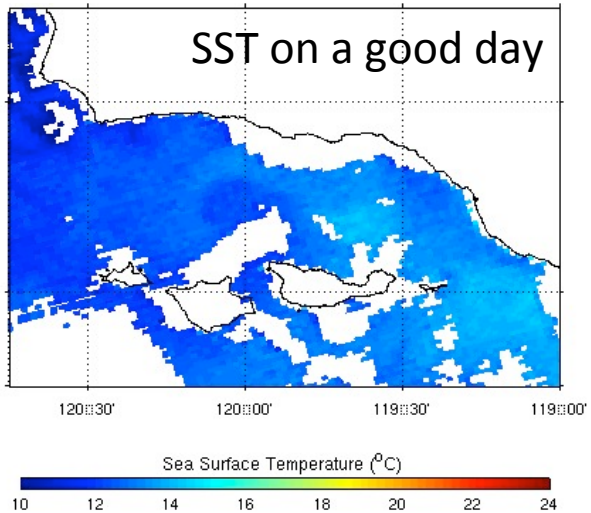


Motivation: MR in Observing System

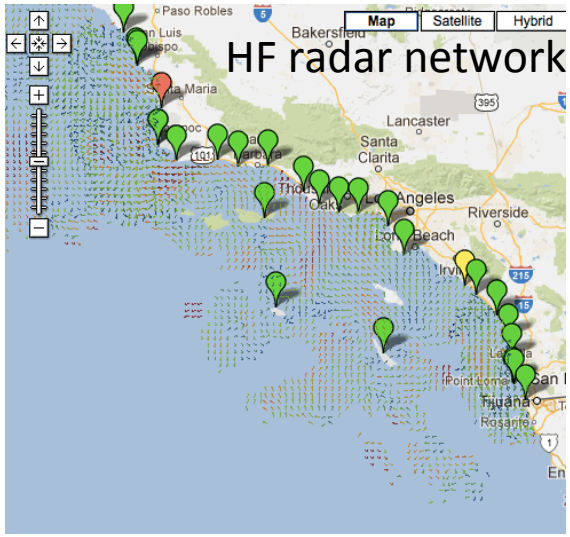
- Spatial density of observing system varies from one obs type to another
  - Dense network
    - Satellite images (SST) at surface
    - HR radar (surface velocity)

$$\mathbf{y}_D = \mathbf{H}_D \mathbf{x} = \mathbf{H}_D(\mathbf{x}_L + \mathbf{x}_S) \quad \text{with } \mathbf{R}_D$$

large scale + small scale



$$\Rightarrow \begin{pmatrix} \text{large scale} \\ \text{small scale} \end{pmatrix} = \begin{pmatrix} \mathbf{y}_{D,L} \\ \mathbf{y}_{D,S} \end{pmatrix} = \begin{pmatrix} \mathbf{H}_D \mathbf{x}_L \\ \mathbf{H}_D \mathbf{x}_S \end{pmatrix} \quad \text{with} \begin{pmatrix} \mathbf{R}_{D.L} \\ \mathbf{R}_{D.S} \end{pmatrix}$$



Approach:

» Assimilate $\mathbf{y}_{D,L}$ and $\mathbf{y}_{D,S}$ separately for $\mathbf{x}_L$ and $\mathbf{x}_S$

MS-MR Formulation for Observation

- MS-MR Variation for $\Delta\mathbf{x}^a = \Delta\mathbf{x}^a{}_L + \Delta\mathbf{x}^a{}_S$
  - Observation

$$\mathbf{y} = \begin{pmatrix} \mathbf{y}_C \\ \mathbf{y}_{D.L} \\ \mathbf{y}_{D.S} \end{pmatrix} = \begin{pmatrix} \mathbf{H}_C(\mathbf{x}_L + \mathbf{x}_S) \\ \mathbf{H}_D\mathbf{x}_L \\ \mathbf{H}_D\mathbf{x}_S \end{pmatrix} \quad \text{with} \quad \begin{pmatrix} \mathbf{R}_C \\ \mathbf{R}_{D.L} \\ \mathbf{R}_{D.S} \end{pmatrix}$$

← Obs that cannot be decomposed

← Obs that can be decomposed

- **Scheme 2. Successive MS-MR VAR from larger to smaller scales**

$$J_L(\Delta\mathbf{x}_L) = \frac{1}{2}(\Delta\mathbf{x}_L)^T(\mathbf{P}_L^b)^{-1}\Delta\mathbf{x}_L + \frac{1}{2}(\mathbf{d}_C - \mathbf{H}_C\Delta\mathbf{x}_L)^T(\mathbf{R}_C + \mathbf{H}_C\mathbf{P}_S^b\mathbf{H}_C{}^T)^{-1}(\mathbf{d}_C - \mathbf{H}_C\Delta\mathbf{x}_L)$$

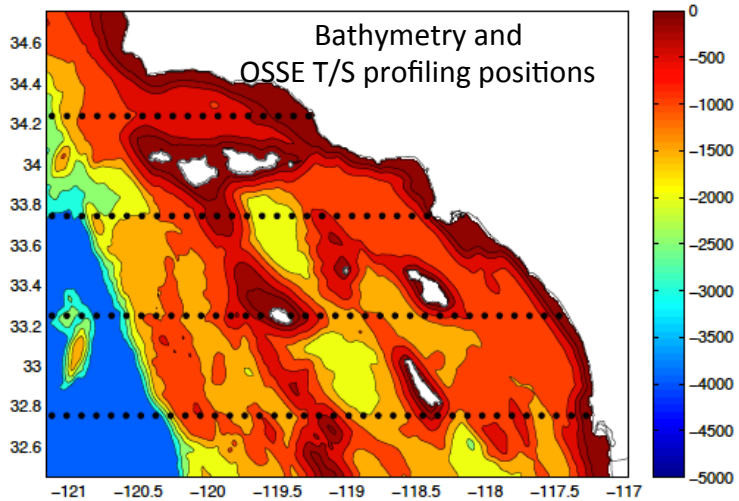
$$+\frac{1}{2}(\mathbf{d}_{D.L} - \mathbf{H}_D\Delta\mathbf{x}_L)^T(\mathbf{R}_{D.L})^{-1}(\mathbf{d}_{D.L} - \mathbf{H}_D\Delta\mathbf{x}_L)$$

$$J_S(\Delta\mathbf{x}_S) = \frac{1}{2}(\Delta\mathbf{x}_S)^T(\mathbf{P}_S^b)^{-1}\Delta\mathbf{x}_S + \frac{1}{2}(\mathbf{d}_C^* - \mathbf{H}_C\Delta\mathbf{x}_S)^T(\mathbf{R}_C + \mathbf{H}_C\mathbf{P}_L^a\mathbf{H}_C{}^T)^{-1}(\mathbf{d}_C^* - \mathbf{H}_C\Delta\mathbf{x}_S)$$

$$+\frac{1}{2}(\mathbf{d}_{D.S}^* - \mathbf{H}_D\Delta\mathbf{x}_S)^T(\mathbf{R}_{D.S}^*)^{-1}(\mathbf{d}_{D.S}^* - \mathbf{H}_D\Delta\mathbf{x}_S)$$

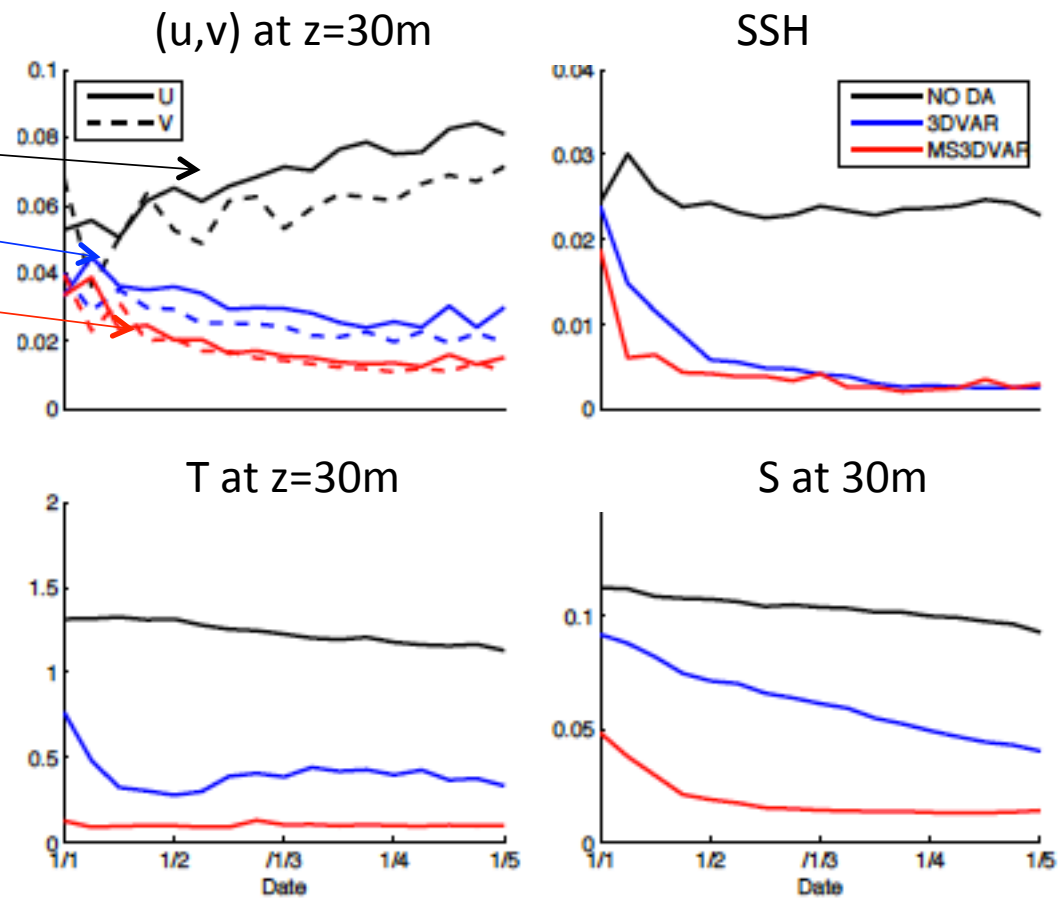
California Coastal Ocean Data Assimilation System

- Observing System Simulation Experiments (OSSEs) Setup
  - Model: Regional Ocean Modeling System (ROMS)
    - Resolution: 1km x 40 levels nested in low-resolution model
    - Atmos forcing: WRF at 2km
  - Southern California Coastal Ocean Observing System (SCCOS)
    - Dense
      - » SST at 2km resolution
      - » Surface (u,v) at 2km resolution
    - Coarse T/S profiling along 4 tracks at
      - » 60km<D_L separation between tracks
      - » 10km<D_s, D_{3Dvar} along track
  - DA systems
    - MS with (D_L,D_S)=(65km,11km)
    - SS with D_S
    - No DA: benchmark



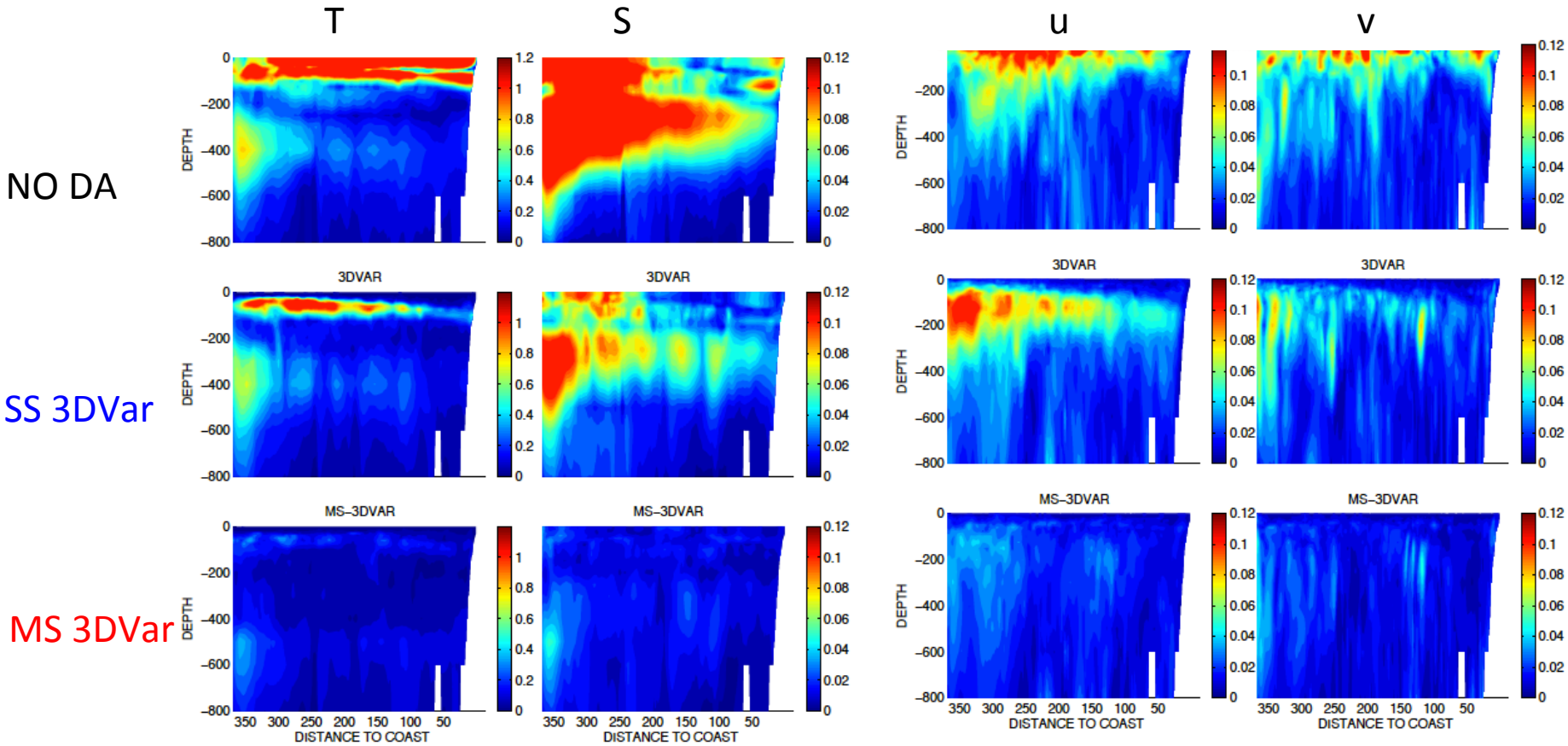
OSSE: RMSE Analysis Error in Time

- Comparison between
  - NO DA
  - SS 3DVar (D_s in $\mathbf{P}^b$)
  - MS 3DVar ($\mathbf{P}^b$ & $\mathbf{H}$)
    - Spin-up faster and converges to smaller RMSE



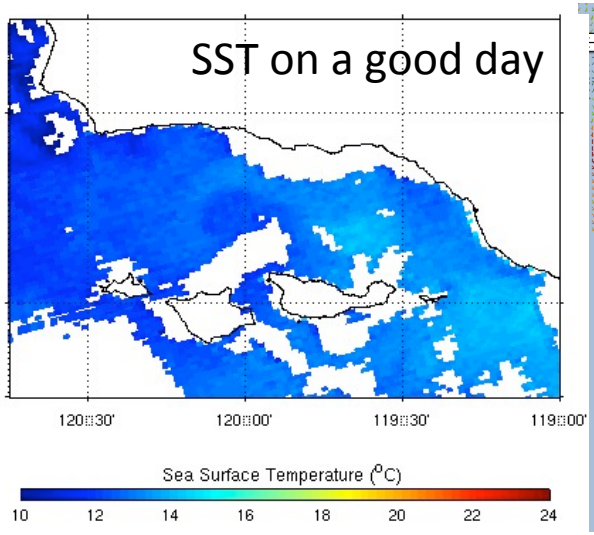
OSSE: RMSE Analysis Error

- Vertical distribution of analysis RMSE At Day 3 (along-shore average)

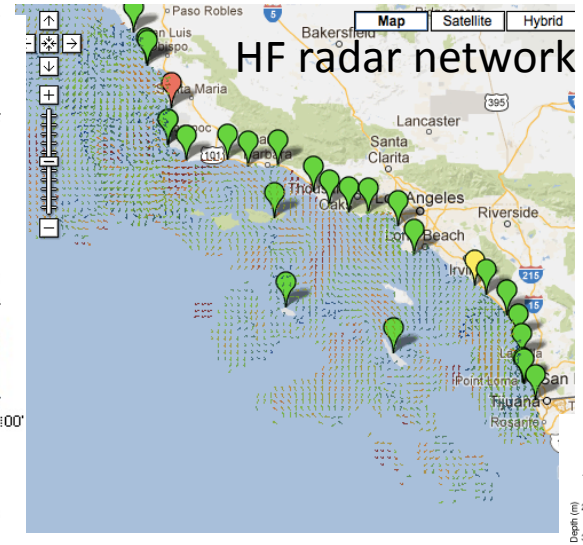


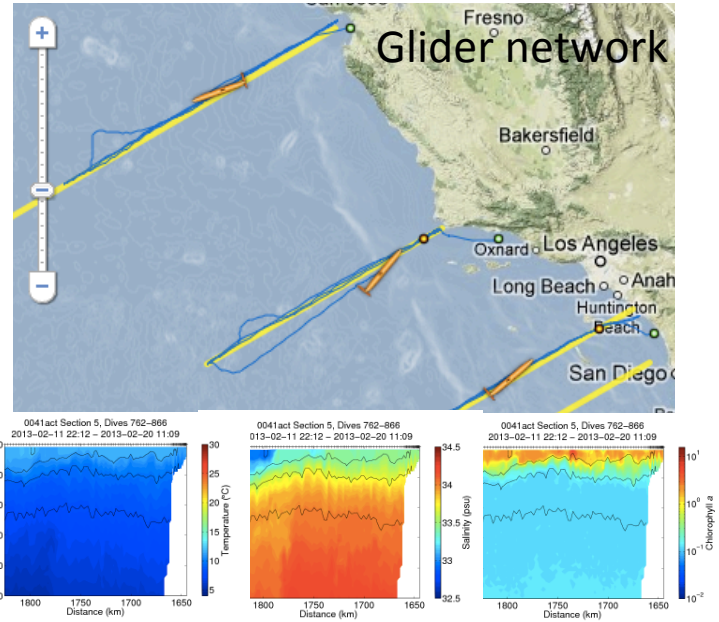
California Coastal Ocean Data Assimilation System

- Real Observation Experiments
  - Initialization: 01/01/2008
  - Observing system (H)



http://sccoos.org

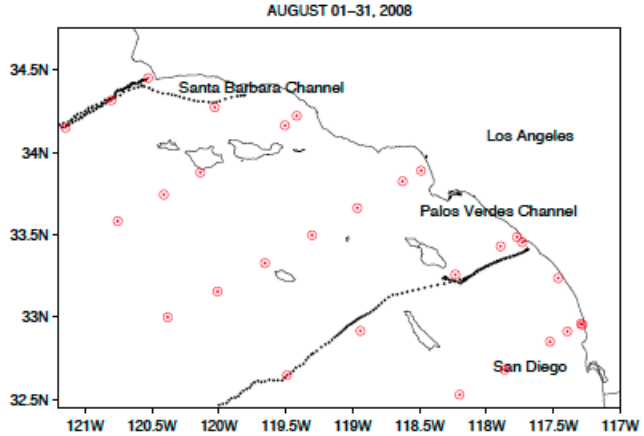




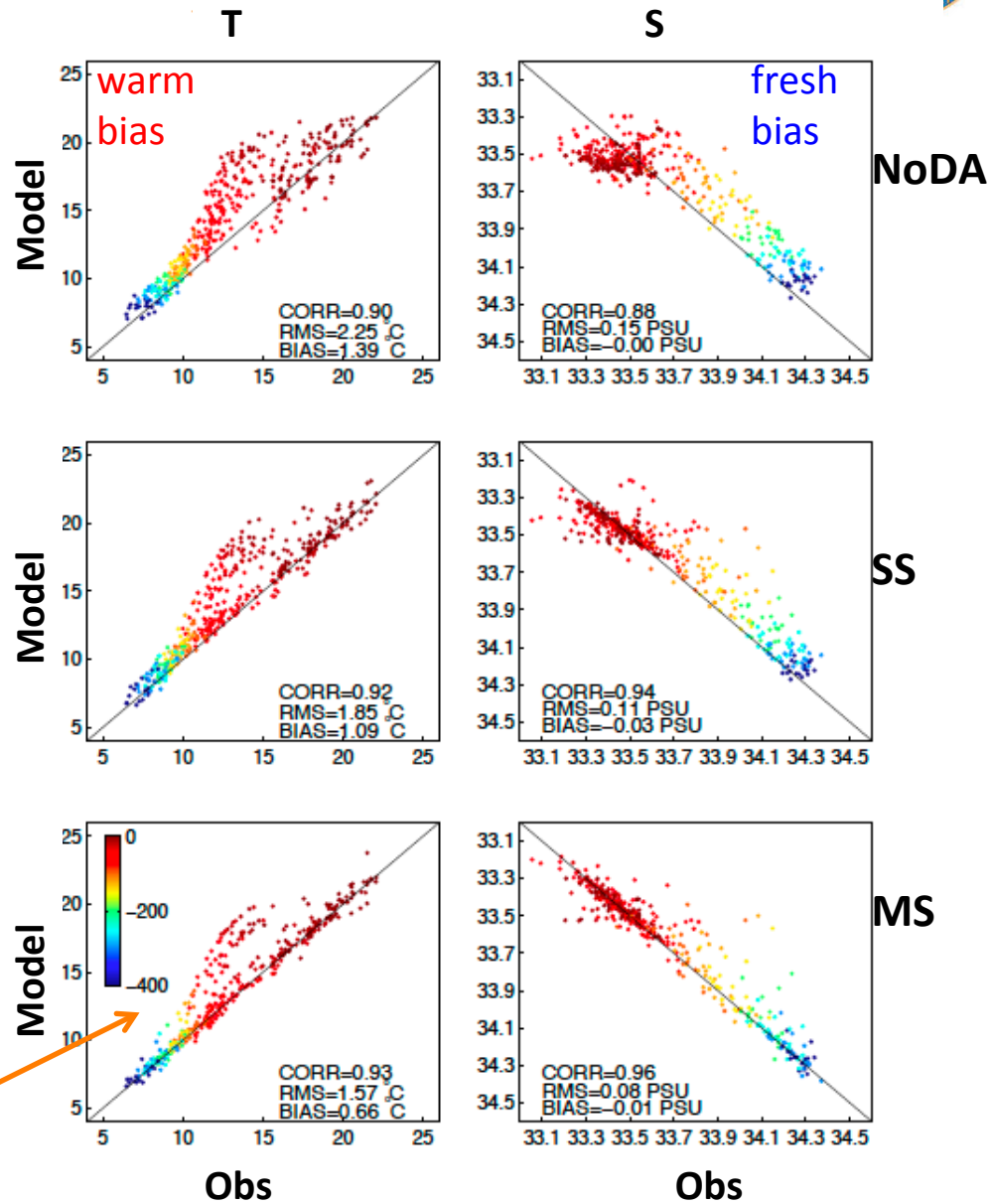
- Performance: Comparison against independent data for bias
  - No DA
  - Standard 3DVar
  - MS 3DVar

California Coastal Ocean Data Assimilation System

- CalCOFI: independent data [08/14-29/2008]
  - CalCOFI location (o)



- MS 3DVar
  - Reduction of error
  - Reduction of bias
- In general
  - More observations needed



Concluding Remarks

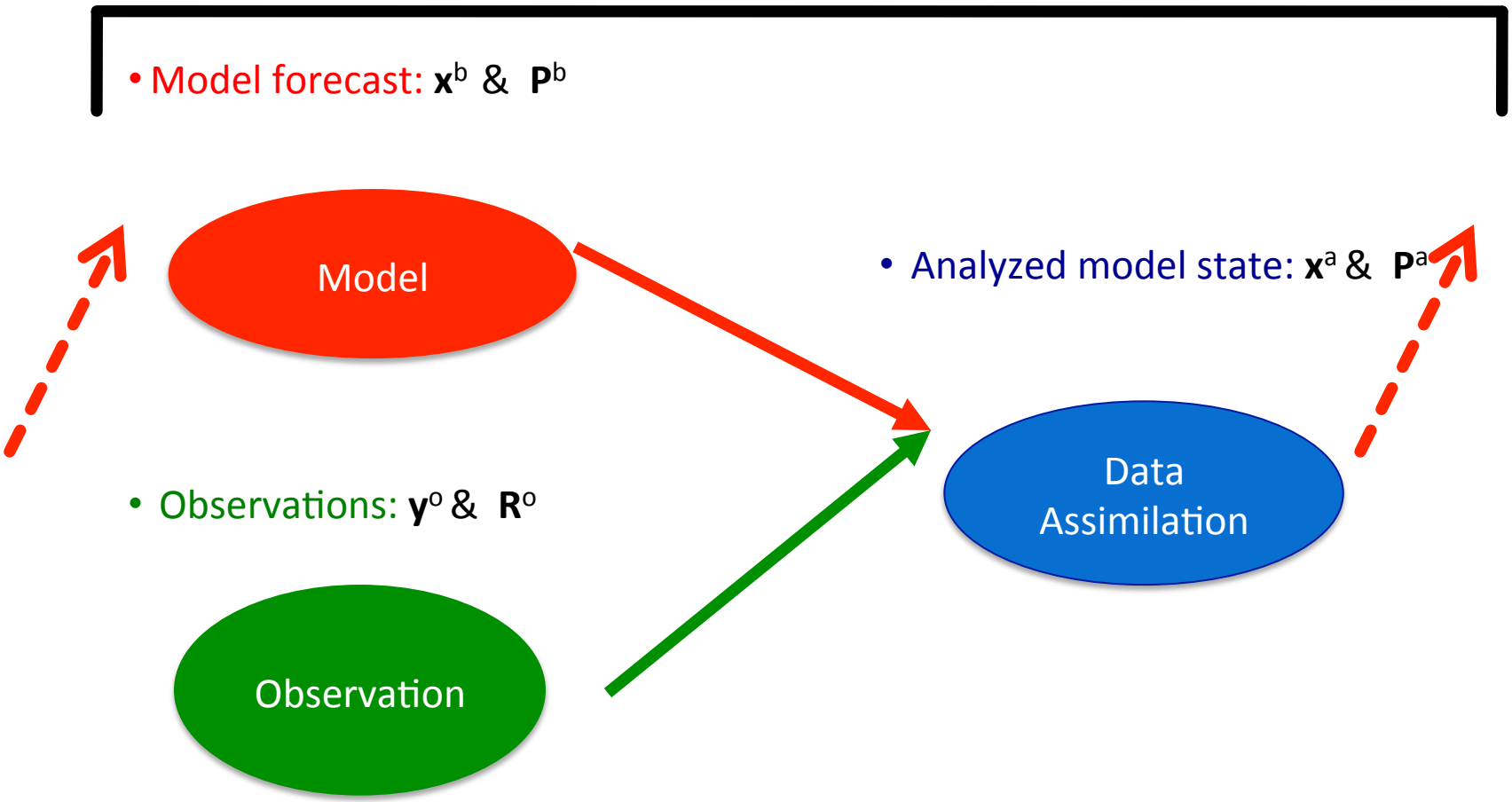
- MS-MR data assimilation is formulated
  - Works very well!
  - Technical Issues
    - Cross-scale correlation can be incorporated
    - Scale separation from dynamics (i.e., Rossby deformation radius)
    - Addressing (cross-)correlation in $\mathbf{R}_L$ and $\mathbf{R}_S$

- Examples
  - SCB: OSSE & Real data experiments
  - Simple 1D example demonstration

- Applications: MS-MR LETKF & Hybrid:
  - Testing on Lorenz model
  - Being implemented SCB … cross-correlation?

Schematic of Data Assimilation: Assimilation Window

- Data assimilation is a iterative method that iterates the cycle over a window:



- Lots of room & serious needs for improvement for ocean data assimilation
- Satellite observations will play role –

Thank you